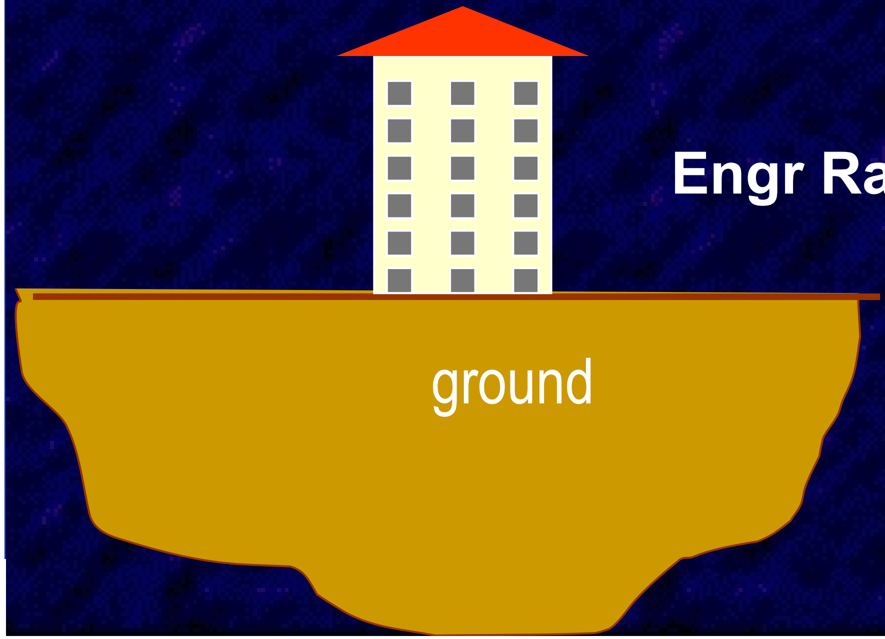
Soil Mechanics-II

STRESS DISTRIBUTION IN SOILS DUE TO SURFACE LOADS



Engr Rameez Sohail

Importance of stresses in soil due to external loads.

- Prediction of settlements of
  - buildings,
  - bridges,
  - Embankments
- Bearing capacity of soils
- Lateral Pressure.

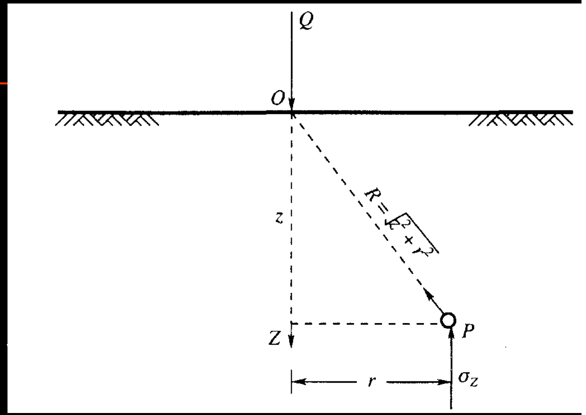
THEORY OF ELASTICITY

- In Engineering mechanics, strain is the ratio of deformation to length and has nothing to do with working out. In an elastic material such as steel, strain is proportional to stress, which is why spring scales work.
- Soil is not an ideal elastic material, but a nearly linear stress-strain relationship exists with limited loading conditions.
- A simplification therefore is made that under these conditions soil can be treated mathematically during vertical compression as an elastic material. (The same assumption frequently is made in finite element analyses.)

- Soil is considered "quasi-elastic," or is described as exhibiting "near-linear elastic behavior.
- There is a limit to near-linear elastic behavior of soils as loading increases and shearing or slipping between individual soil particles increases.
- When that happens any semblance to an elastic response is lost as shearing more closely simulates plastic behavior.
- This is the behavioral mode of soils in landslides, bearing capacity failures, and behind most retaining walls

- The extent of the elastic layer below the surface loadings may be any one of the following:
    - Infinite in the vertical and horizontal directions.
    - Limited thickness in the vertical direction underlain with a rough rigid base such as a rocky bed.
- The loads at the surface may act on flexible or rigid footings. The stress conditions in the elastic layer below vary according to the rigidity of the footings and the thickness of the elastic layer.
- All the external loads considered are vertical loads only as the vertical loads are of practical importance for computing settlements of foundations.

BOUSSINESCTS FORMULA FOR POINT LOADS



- A semi-infinite solid is the one bounded on one side by a horizontal surface, here the surface of the earth, and infinite in all the other directions. The problem of determining stresses at any point *P at a depth z as* a result of a surface point load was (1885) on the following assumptions.
- The soil mass is elastic, isotropic (having identical properties in all direction throughout), homogeneous (identical elastic properties) and semi-infinite.
- The soil is weightless.
- The load is a point load acting on the surface. vertical stress бz, at point *P* under point load Q is given as

$$\sigma_z = \frac{3Q}{2\pi z^2} \frac{1}{[1 + (r/z)^2]^{5/2}} = \frac{Q}{z^2} I_B$$

- where, *r = the horizontal distance between an arbitrary point P below the surface and the vertical* axis through the point load *Q.*
- *z = the vertical depth of the point P from the surface.*
- *I_B - Boussinesq stress coefficient =*

$$= \frac{3}{2\pi} \frac{1}{[1 + (r/z)^2]^{5/2}}$$

- The values of the Boussinesq coefficient *I_B can be determined for a number of values of r/z.* The variation of I_B with *r/z in a graphical form is given in Fig.*

Slution:

$$\sigma_z = \frac{Q}{z^2} I_B, \text{ where } I_B = \frac{3/2\pi}{\left[1+(r/z)^2\right]^{5/2}}$$

(i) When $r/z = 0$, $I_B = 3/2\ \pi = 0.48$, $\sigma_z = 0.48\frac{Q}{z^2} = 0.48 \times \frac{1000}{4 \times 4} = 30\ \text{kN/m}^2$

(ii) When $r/z = 3/4 = 0.75$

$$I_B = \frac{3/2\pi}{\left[1+(0.75)^2\right]^{5/2}} = 0.156,\ \ \sigma_z = \frac{0.156 \times 1000}{4 \times 4} = 9.8\ \text{kN/m}^2$$

Problem

- A concentrated load of 1000 kN is applied at the ground surface. Compute the vertical pressure
  - (i) at a depth of 4 m below the load,
  - (ii) at a distance of 3 m at the same depth. Use Boussinesq's equation.
- Solve your self.

WESTERGAARD'S FORMULA FOR POINT LOADS

- Actual soil is neither isotropic nor homogenous.
- Westergaard, a British Scientist, proposed (1938) a formula for the computation of vertical stress $б_z$ *by a point load, Q, at the surface as*

$$\sigma_z = \frac{Q}{2\pi z^2}\frac{\sqrt{(1-2\mu)/(2-2\mu)}}{[(1-2\mu)/(2-\mu)+(r/z)^2]^{3/2}} = \frac{Q}{z^2}I_w$$

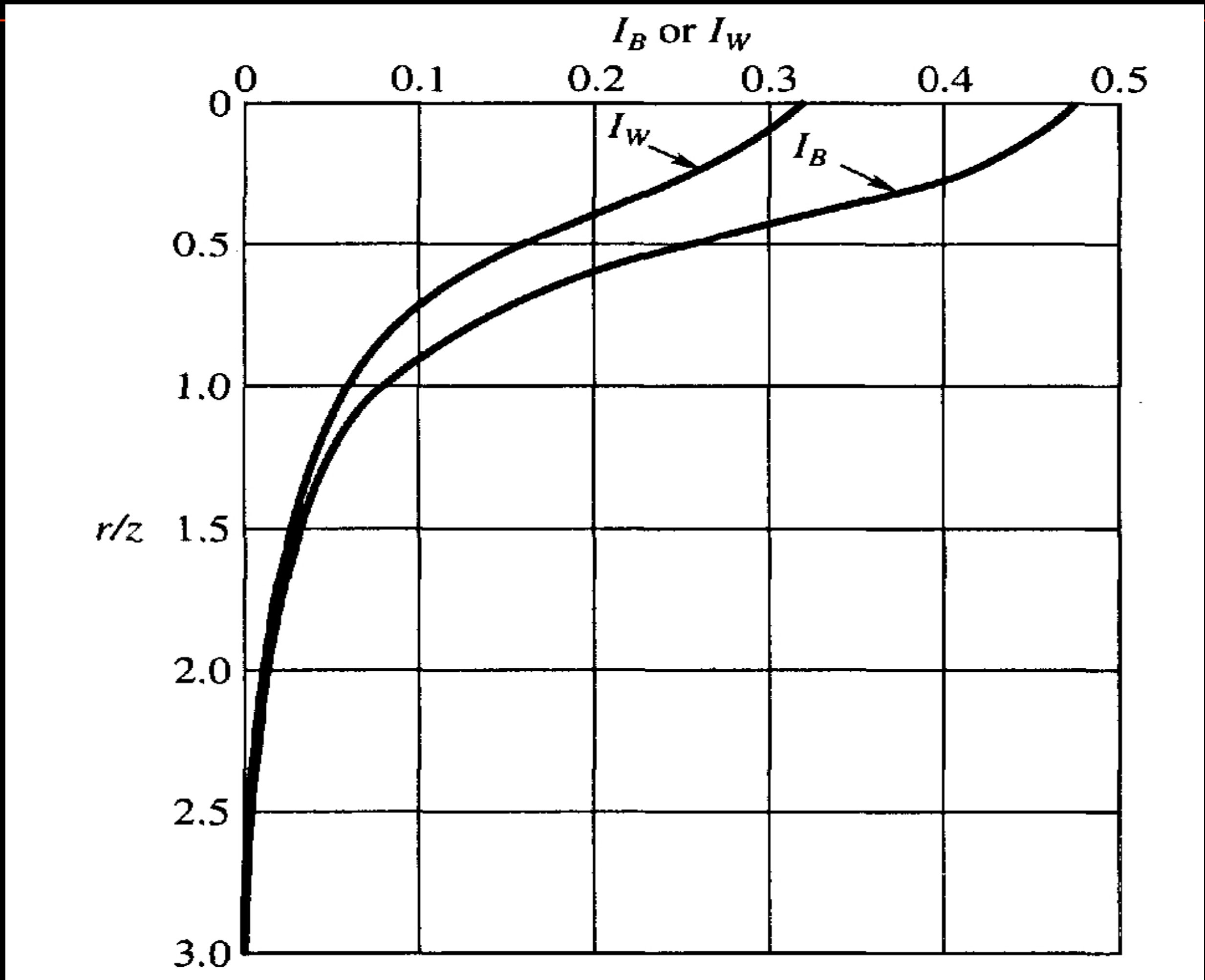
- in which *μ, is Poisson's ratio. If μ, is taken as zero for all practical purposes,*

$$\sigma_z = \frac{Q}{\pi z^2}\frac{1}{[1+2(r/z)^2]^{3/2}} = \frac{Q}{z^2}I_w$$

where $I_w = \frac{(1/\pi)}{[1+2(r/z)^2]^{3/2}}$ is the Westergaard stress coefficient.

- The variation of I_B with the ratios of *(r/z) is shown graphically on next slide along with the Boussinesq's coefficient IB. The value* of I_w *at r/z = 0 is 0.32 which is less than that of IB by 33 per cent.*
- Geotechnical engineers prefer to use Boussinesq's solution as this gives conservative results.

Values of I_B or I_w for use in the Boussinesq or Westergaard formula



Problem: Solve in the class

- A concentrated load of 45000 Ib acts at foundation level at a depth of 6.56 ft below ground surface.
- Find the vertical stress along the axis of the load at a depth of 32.8 ft and at a radial distance of
- 16.4 ft at the same depth by
- (a) Boussinesq, and
- (b) Westergaard formulae for μ= 0.
- Neglect the depth of the foundation.

(a) Boussinesq Eq. (6.1a)

$$\sigma_z = \frac{Q}{z^2} I_B, \quad I_B = \frac{3}{2\pi} \left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

Substituting the known values, and simplifying

$$I_B = 0.2733 \text{ for } r/z = 0.5$$

$$\sigma_z = \frac{45000}{(32.8)^2} \times 0.2733 = 11.43 \text{ lb/ft}^2$$

(b) Westergaard (Eq. 6.3)

$$\sigma_z = \frac{Q}{z^2} I_w, \quad I_w = \frac{1}{\pi}\left[\frac{1}{1+2(r/z)^2}\right]^{3/2}$$

Substituting the known values and simplifying, we have,

$$I_w = 0.1733 \text{ for } r/z = 0.5$$

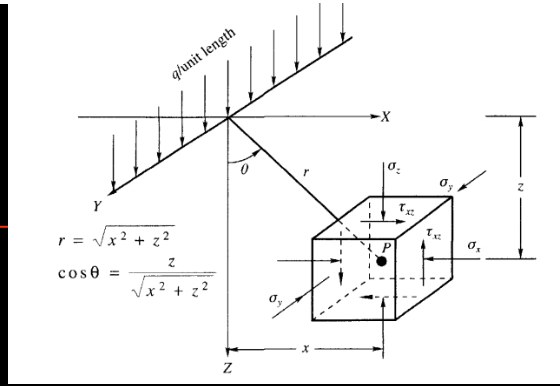
therefore,

$$\sigma_z = \frac{45000}{(32.8)^2} \times 0.1733 = 7.25 \text{ lb/ft}^2$$

Home Assignment: Example 6.3

- A rectangular raft of size 30 x 12 m founded at a depth of 2.5 m below the ground surface is subjected to a uniform pressure of 150 kPa. Assume the center of the area is the origin of coordinates (0, 0). and the corners have coordinates (6, 15).
- Calculate stresses at a depth of 20 m below the foundation level by the methods of (a) Boussinesq, and (b) Westergaard at coordinates of
- (0, 0), (0, 15), (6, 0) (6, 15) and (10, 25).
- Also determine the ratios of the stresses as obtained by the two methods. Neglect the effect of foundation depth on the stresses.

LINE LOADS



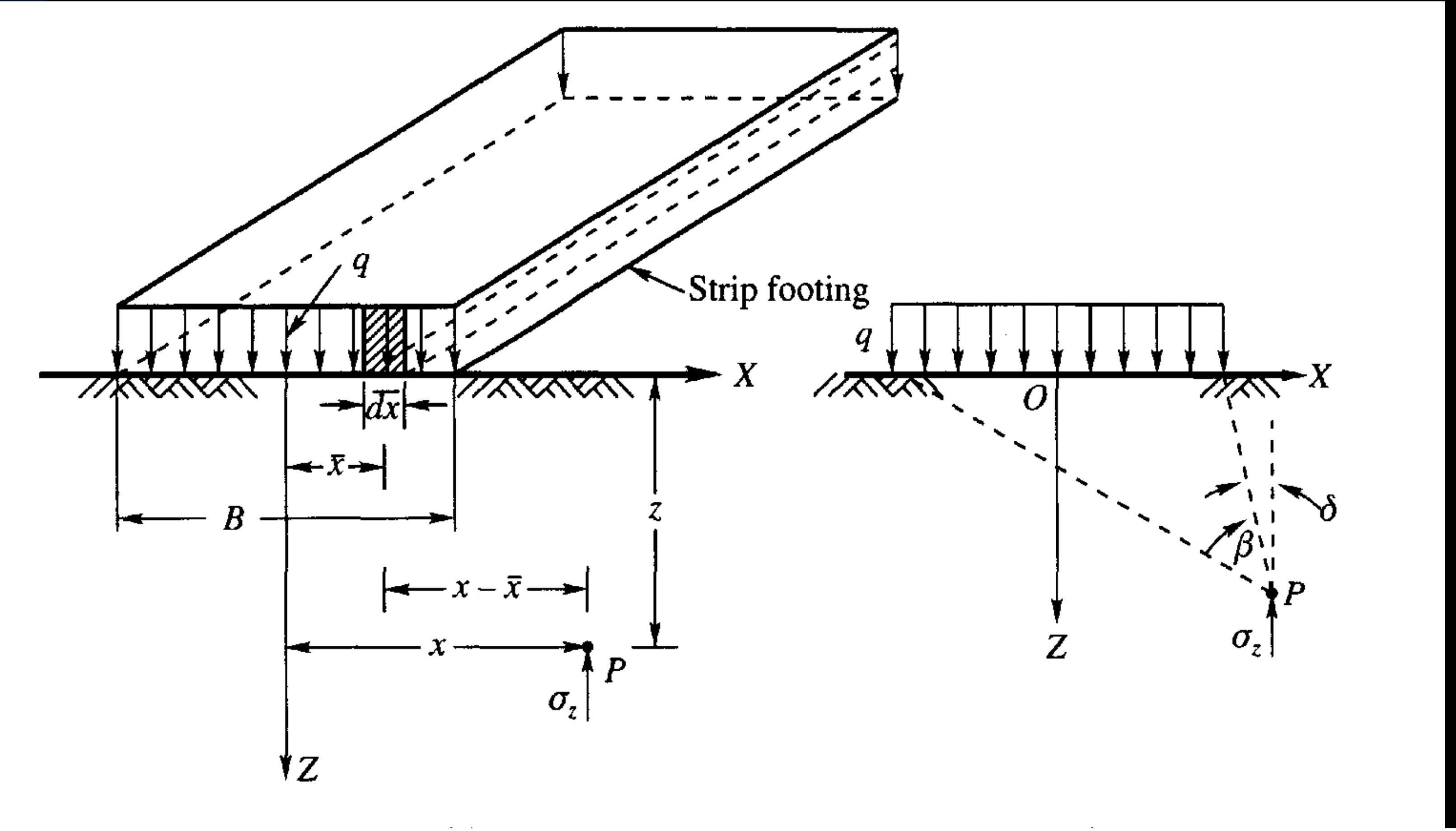
- By applying the principle of the above theory, the stresses at any point in the mass due to a line load of infinite extent acting at the surface may be obtained.
- The state of stress encountered in this case is that of a plane strain condition. The strain at any point *P in the Y*-direction parallel to the line load is assumed equal to zero. The stress $б_y$ normal to the XZ-plane is the same at all sections and the shear stresses on these sections are zero.
- The vertical $б_z$ stress *at point P may be written in* rectangular coordinates as

$$\sigma_z = \frac{q}{z}\frac{2/\pi}{[1+(x/z)^2]^2} = \frac{q}{z} I_z$$

- where, I_z is the influence factor equal to 0.637 at *x/z =0.*

STRIP LOADS

- Such conditions are found for structures extended very much in one direction, such as strip and wall foundations, foundations of retaining walls, embankments, dams and the like.



- Fig. shows a load *q per unit area acting on a strip of infinite length and of constant* width *B. The vertical stress at any arbitrary point P due to a line load of qdx acting at* $x = \bar{x}$ *can be* written from Eq. as

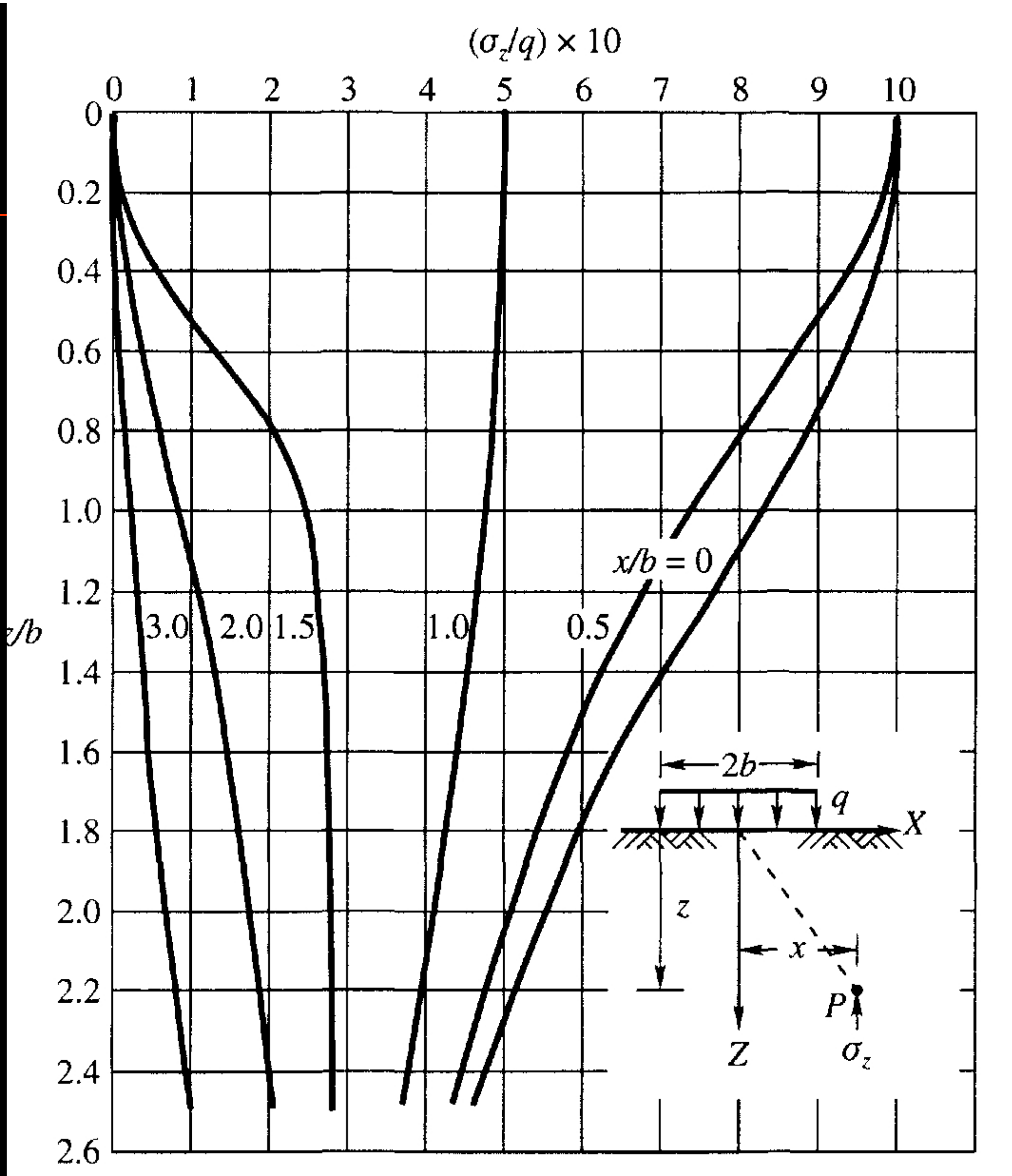
$$d\sigma_z = \frac{2q}{\pi} \frac{z^3}{[(x-\bar{x})^2 + z^2]^2}$$

- Applying the principle of superposition, the total stress бz at point *P due to a strip load* distributed over a width *B(= 2b) may be written as*

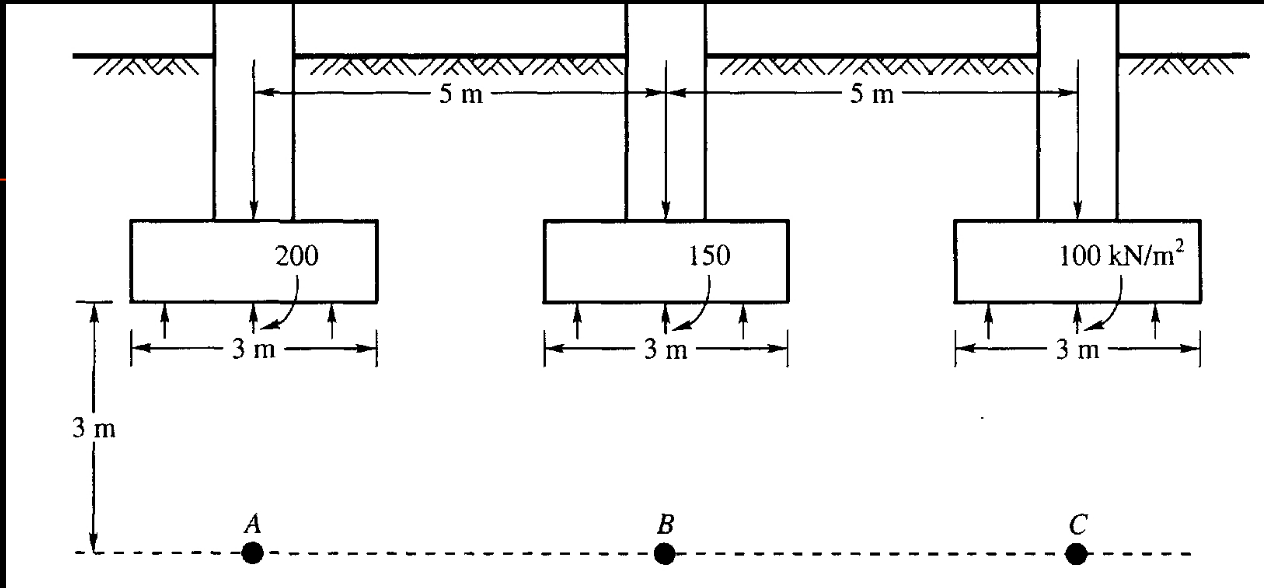
$$\sigma_z = \frac{2q}{\pi} \int_{-b}^{+b} \frac{z^3}{[(x-\bar{x})^2 + z^2]^2} dx$$

or $$\sigma_z = \frac{q}{\pi} \tan^{-1} \frac{z}{x-b} - \tan^{-1} \frac{z}{x+b} - \frac{2bz(x^2 - b^2 - z^2)}{(x^2 - b^2 + z^2)^2 + 4b^2z^2}$$

- The non-dimensional values *can be* expressed in a more convenient form as



Non-dimensional values of σ_z/q for strip load



- **Example 6.4**
- Three parallel strip footings 3 m wide each and 5 m apart center to center transmit contact pressures of 200, 150 and 100 kN/m2 respectively.
- Calculate the vertical stress due to the combined loads beneath the centers of each footing at a depth of 3 m below the base. Assume the footings are placed at a depth of 2 m below the ground surface. Use Boussinesq's method for line loads.

- We know $\sigma_z = \frac{q}{z}\frac{2/\pi}{\left[1+(x/z)^2\right]^2} = \frac{q}{z} I_z$

The stress at A (Fig. Ex. 6.4) is

$$(\sigma_z)_A = \frac{2\times 200}{3.14\times 3}\left[\frac{1}{1+(0/3)^2}\right]^2 + \frac{2\times 150}{3.14\times 3}\left[\frac{1}{1+(5/3)^2}\right]^2$$

$$+\frac{2\times 100}{3.14\times 3}\left[\frac{1}{1+(10/3)^2}\right]^2 = 45 \text{ kN/m}^2$$

The stress at B

$$(\sigma_z)_B = \frac{2\times 200}{3\pi}\left[\frac{1}{1+(5/3)^2}\right]^2 + \frac{2\times 150}{3\pi}\left[\frac{1}{1+(0/3)^2}\right]^2$$

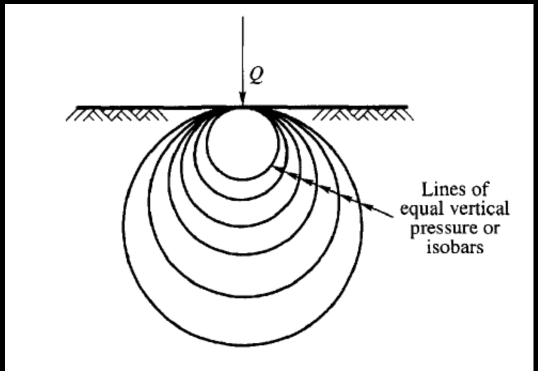
$$+\frac{2\times 100}{3\pi}\left[\frac{1}{1+(5/3)^2}\right]^2 = 36.3 \text{ kN/m}^2$$

The stress at C

$$(\sigma_z)_C = \frac{2\times 200}{3\pi}\ \frac{1}{1+(10/3)^2}\ ^2 + \frac{2\times 150}{3\pi}\ \frac{1}{1+(5/3)^2}\ ^2 + \frac{2\times 100}{3\pi} = 23.74 \text{ kN/m}^2$$

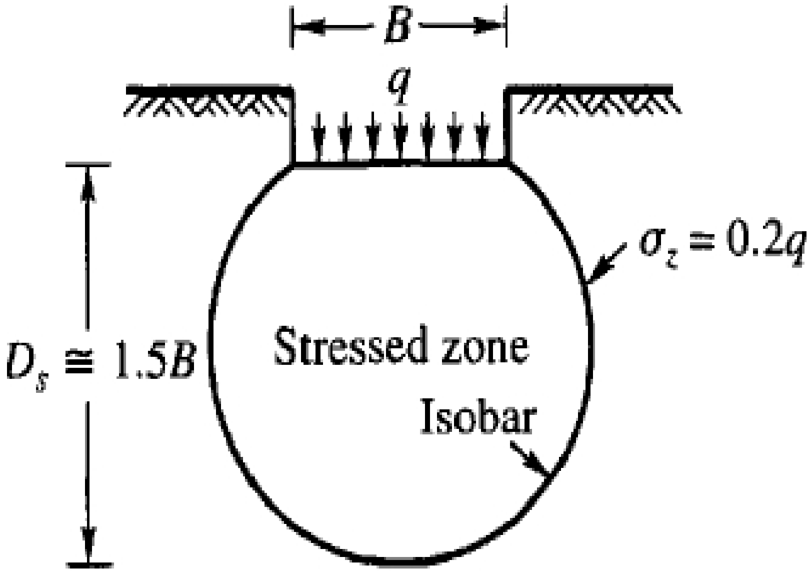
PRESSURE ISOBARS-Pressure Bulb

- An *isobar is a line which connects all points of equal stress below the ground surface. In other* words, an isobar is a stress contour. We may draw any number of isobars as shown in Fig. for any given load system.
- Each isobar represents a fraction of the load applied at the surface. Since these isobars form closed figures and resemble the form of a bulb, they are also termed *bulb of pressure or simply the pressure bulb.*
- *Normally isobars are drawn for vertical, horizontal and shear* stresses. The one that is most important in the calculation of settlements of footings is the vertical pressure isobar.

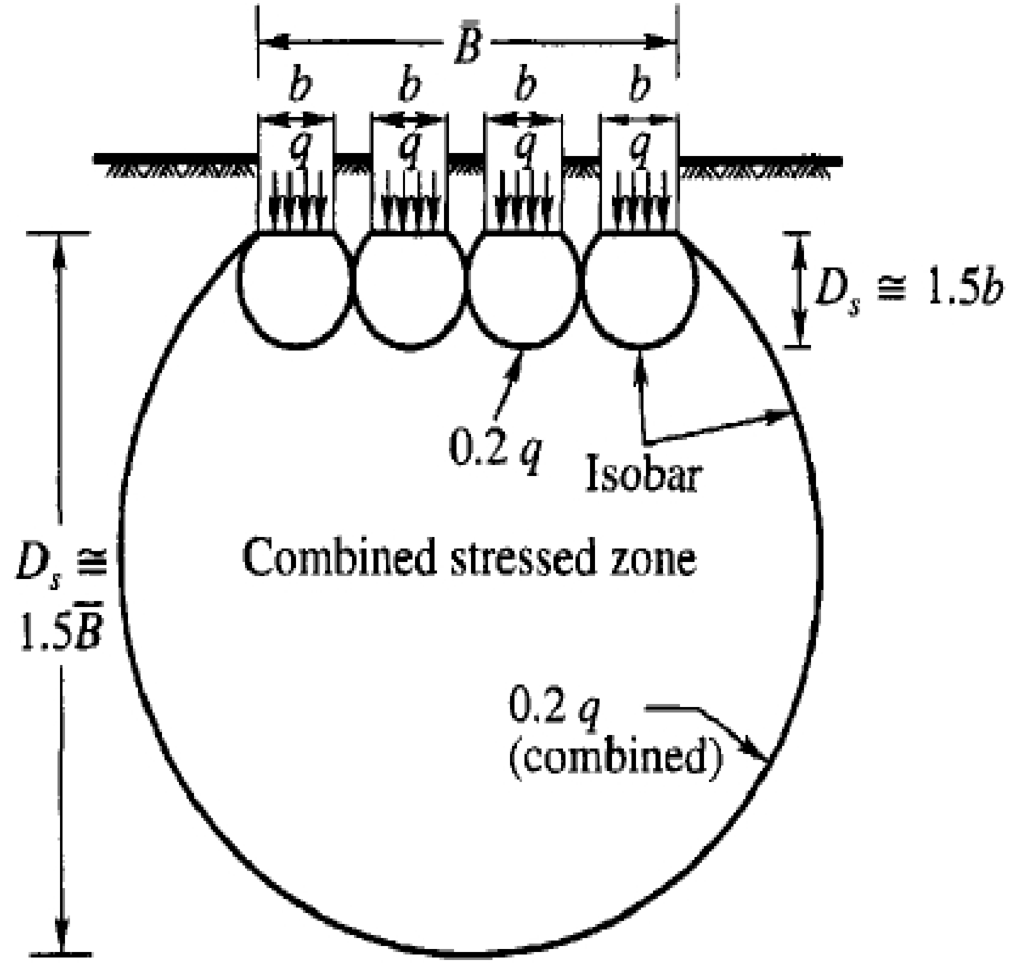


- we may draw any number of isobars for any given load system, but the one that is of practical significance is the one which encloses a soil mass which is responsible for the settlement of the structure.
- The depth of this stressed zone may be termed as the *significant depth Ds which is responsible for the settlement* of the structure. Terzaghi recommended that for all practical purposes one can take a *stress contour* which represents 20 per cent of the foundation contact pressure *q, i.e, equal to 0.2q.*
- Terzaghi's recommendation was based on his observation that direct stresses are considered of negligible magnitude when they are smaller than 20 per cent of the intensity of the applied stress from structural loading, and that most of the settlement, approximately 80 per cent of the total, takes place at a depth less than *Ds.*
- The depth *Ds is approximately equal to 1.5 times the* width of square or circular footings

Significant depths:

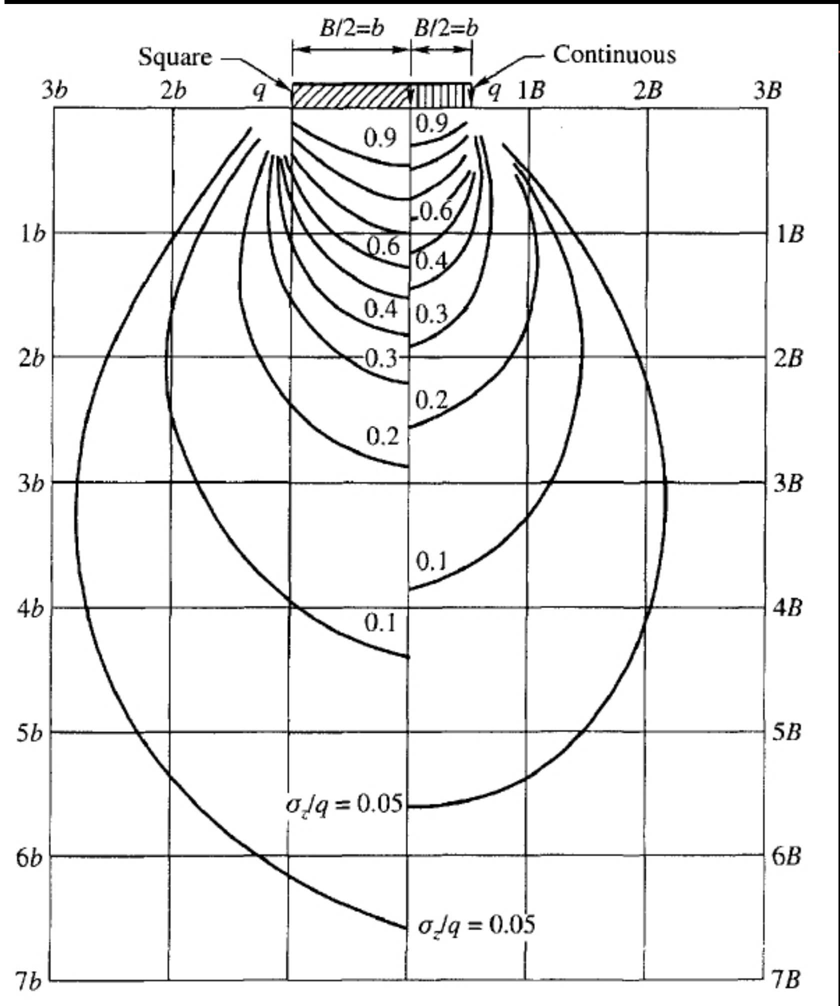


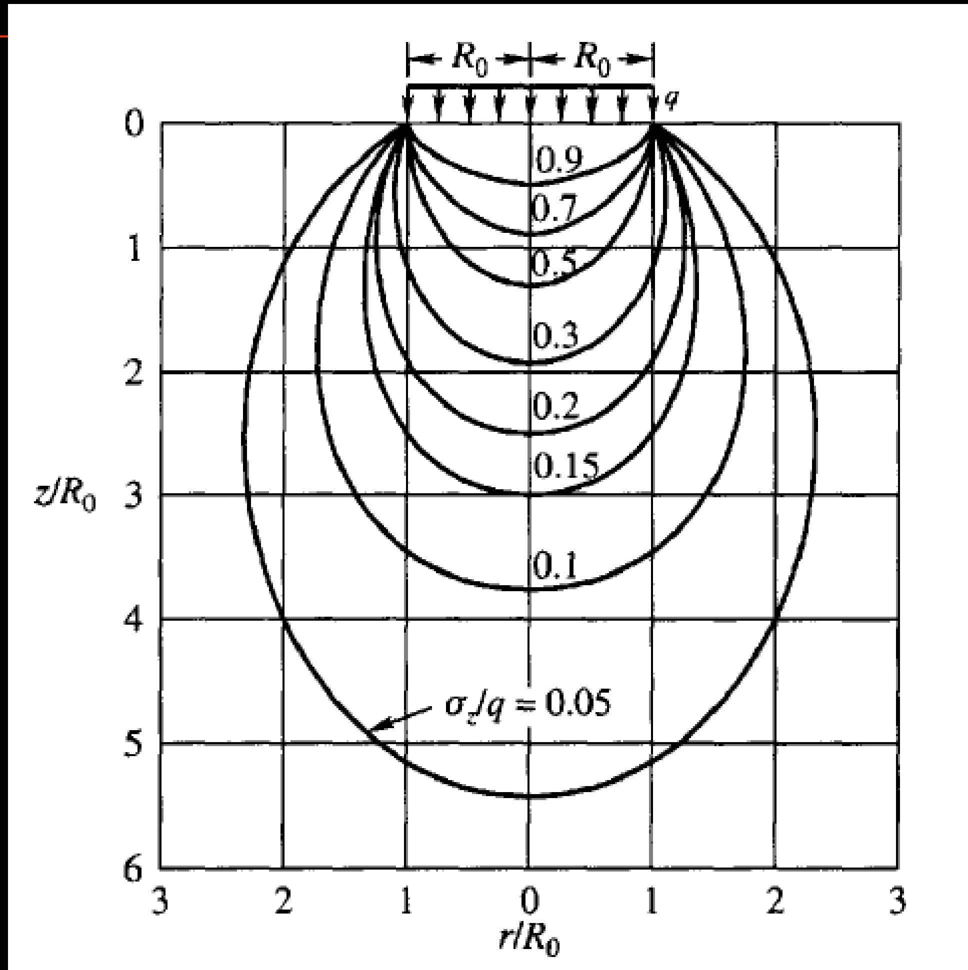
(a) Significant depth of stressed zone for single footing



(b) Effect of closely placed footings

Pressure Isobars for Footings





Example of Pressure bulb.

A single concentrated load of 1000 kN acts at the ground surface. Construct an isobar for $\sigma_z = 40\ kN/m^2$ by making use of the Boussinesq equation.

Solution

From Eq. (6.1a) we have

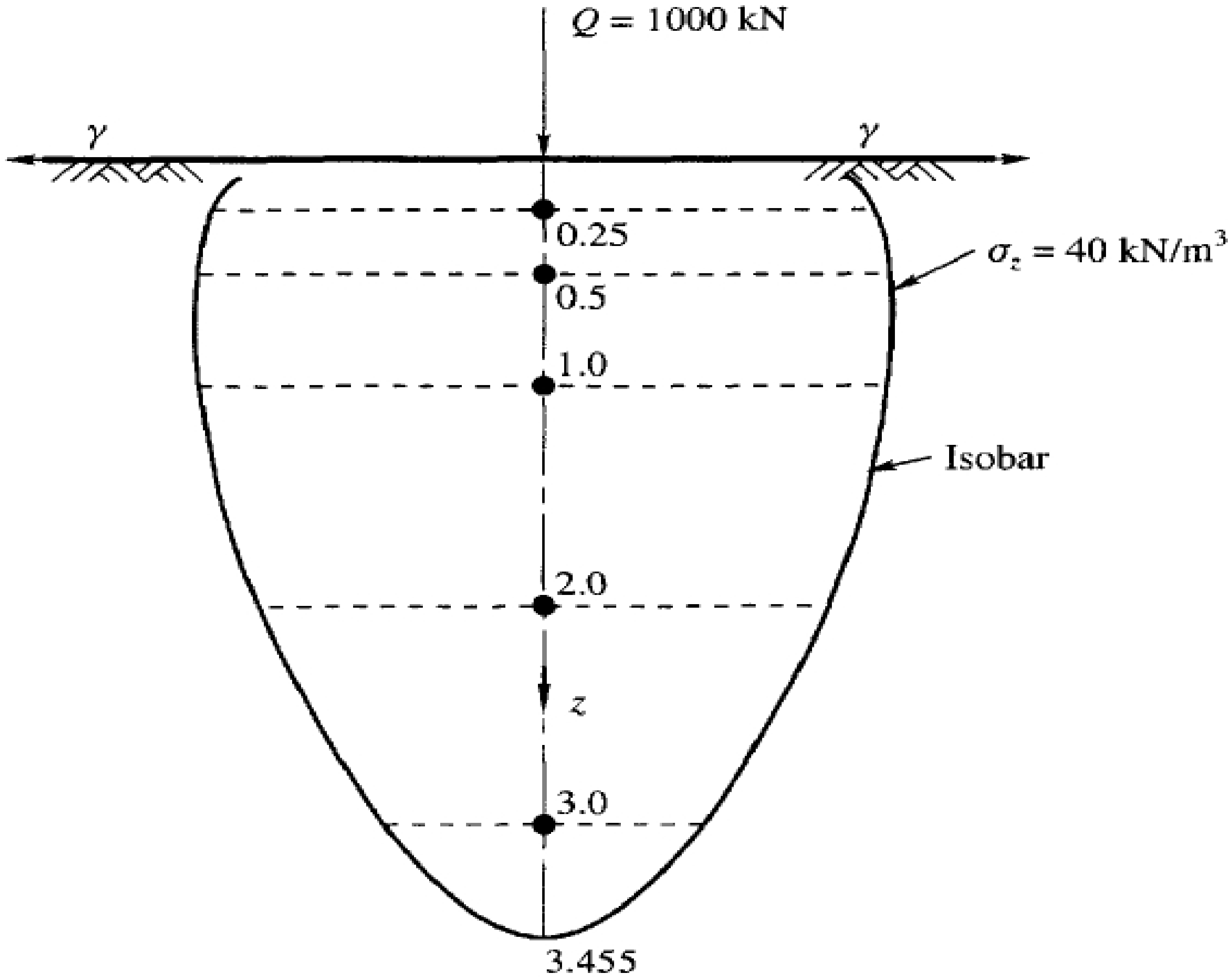
$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

We may now write by rearranging an equation for the radial distance r as

$$r = \sqrt{z}\sqrt{\left(\frac{3Q}{2\pi z^2 \sigma_z}\right)^{2/5} - 1}$$

Now for $Q = 1000$ kN, $\sigma_z = 40\ kN/m^2$, we obtain the values of r_1, r_2, r_3, etc. for different depths z_1, z_2, z_3, etc. The values so obtained are

z (m)	r (m)
0.25	1.34
0.50	1.36
1.0	1.30
2.0	1.04
3.0	0.60
3.455	0.00



Q = 1000 kN
γ
γ
0.25
0.5
1.0
σz = 40 kN/m3
Isobar
2.0
z
3.0
3.455

Home assignment.

1. A column of a building transfers a concentrated load of 225 kips to the soil in contact with the footing. Estimate the vertical pressure at the following points by making use of the Boussinesq and Westergaard equations.

- (i) Vertically below the column load at depths of 5, 10, and 15 ft.
- (ii) At radial distances of 5, 10 and 20 ft and at a depth of 10 ft.

2. A reinforced concrete water tank of size 25 ft x 25 ft and resting on the ground surface carries a uniformly distributed load of 5.25 kips/ft^2. Estimate the maximum vertical pressures at depths of 37.5 and 60 ft by point load approximation below the center of the tank.

- 3. A single concentrated load of 100 Kips acts at the ground surface. Construct an isobar for 1 t/sft by making use of the Westergards,s equation.