10.6 DESIGN AND ANALYSIS OF ECCENTRICALLY LOADED COLUMNS USING INTERACTION DIAGRAMS

If individual column interaction diagrams were prepared as described in the preceding sections, it would be necessary to have a diagram for each different column cross section, for each different set of concrete and steel grades, and for each different bar arrangement. The result would be an astronomical number of diagrams. The number can be tremendously reduced, however, if the diagrams are plotted with ordinates of P_n/f'_cA_g (instead of P_n) and with abscissas of P_ne/f'_cA_gh (instead of M_n). Thus each interaction diagram can be used for cross sections with widely varying dimensions. The ACI has prepared interaction curves in this manner for the different cross section and bar arrangement situations shown in Figure 10.14 and for different grades of steel and concrete.[3]

Two of the ACI diagrams are given in Figures 10.15 and 10.16, while Appendix A (Graphs A.2–A.13) presents several other ones for the situations given in parts (a), (b), and (d) of Figure 10.14. *Notice that these ACI diagrams do not include the three modifications described in the last section.*

The ACI column interaction diagrams are used in Examples 10.3 to 10.7 to design or analyze columns for different situations. In order to correctly use these diagrams, it is necessary to compute the value of γ (gamma), which is equal to the distance from the center of the bars on one side of the column to the center of the bars on the other side of the col-

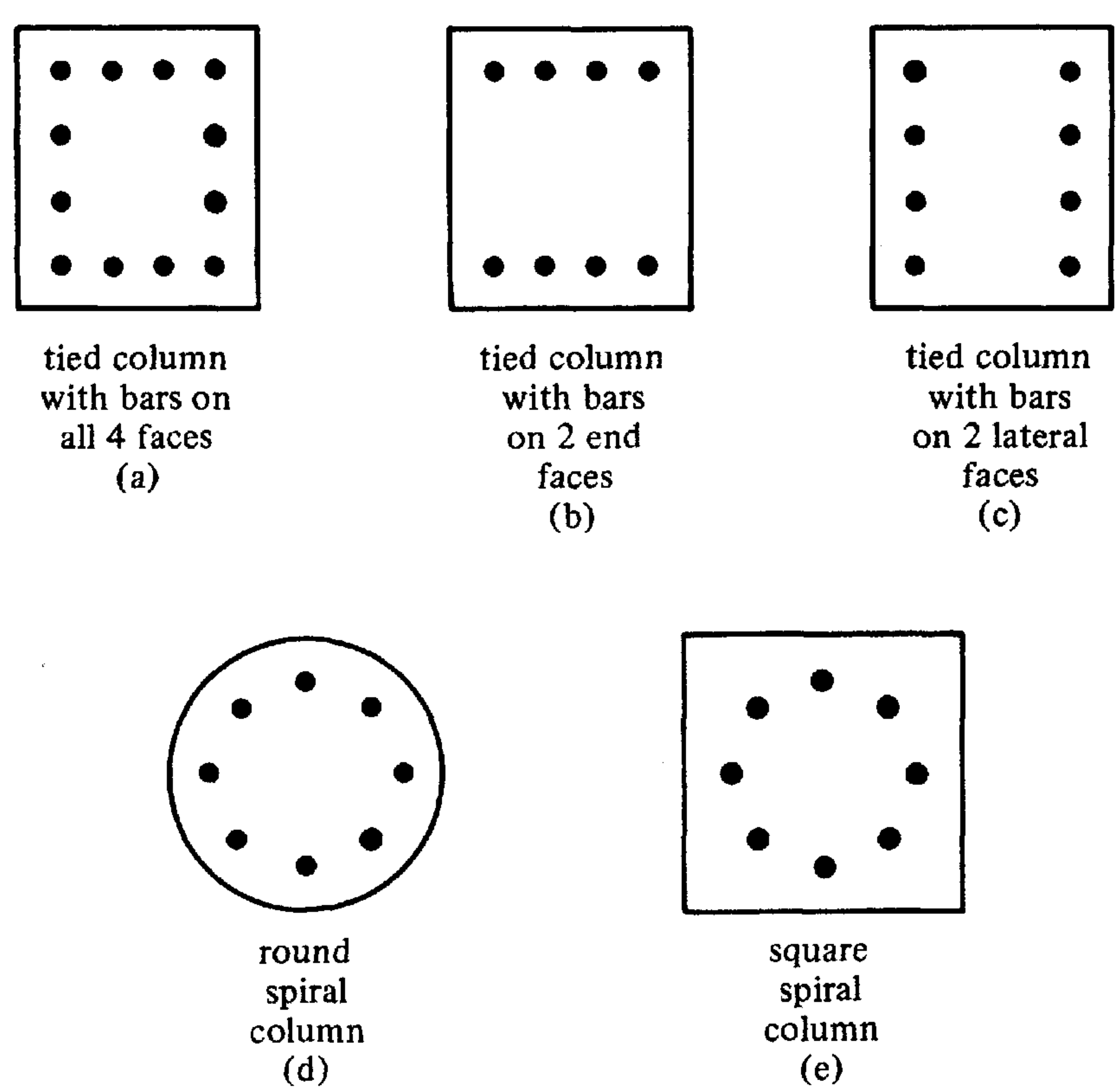

Figure 10.14

[3]American Concrete Institute, *Design Handbook*, 1997, Publication SP-17(97), Detroit, 482 pages.

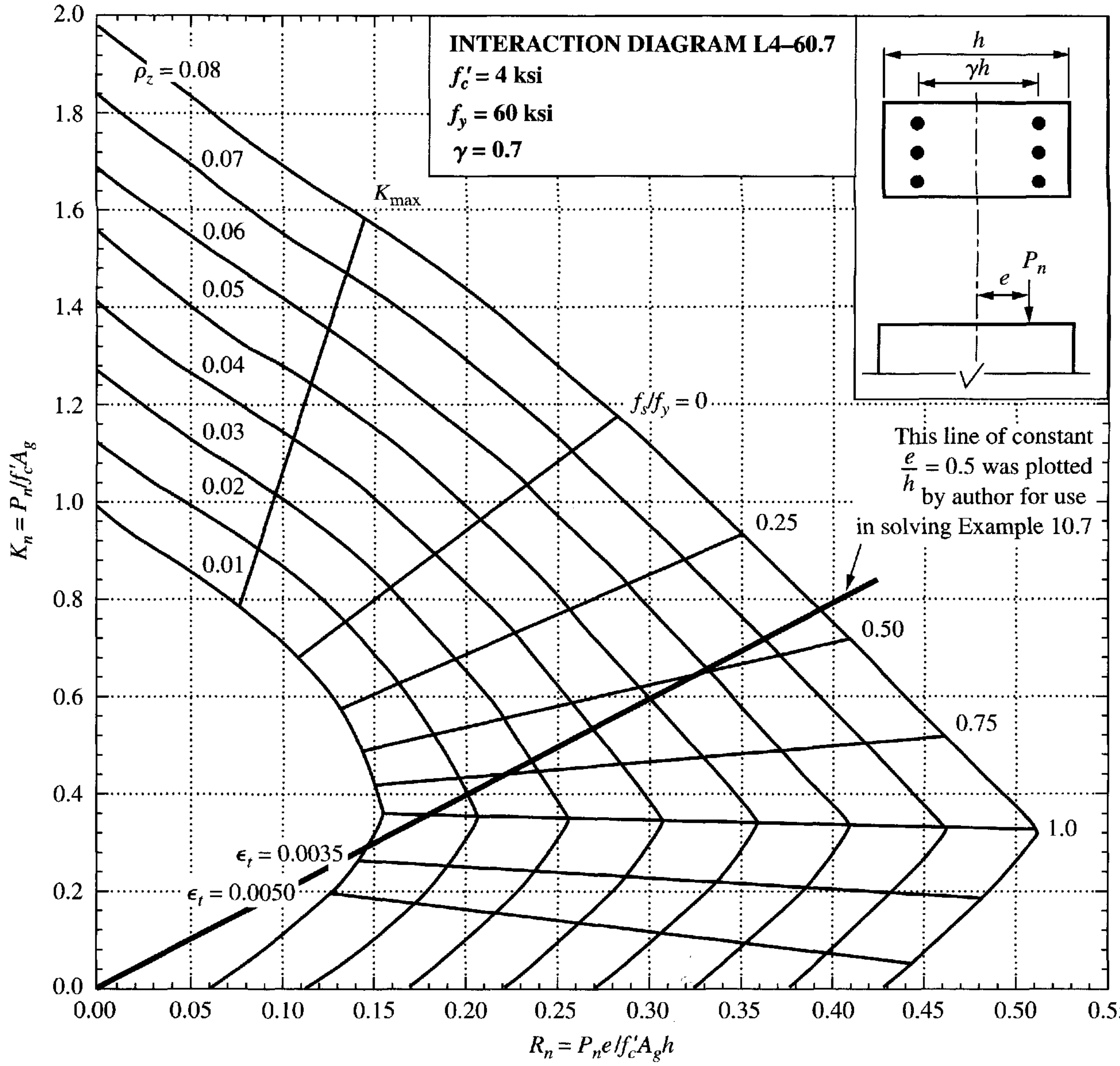

Figure 10.15 ACI rectangular column interaction diagrams when bars are placed on two faces only. (Permission of American Concrete Institute.)

umn divided by h, the depth of the column (both values being taken in the direction of bending). Usually the value of γ obtained falls in between a pair of curves, and interpolation of the curve readings will have to be made.

Warning

Be sure that the column picture at the upper right of the interaction curve being used agrees with the column being considered. In other words, are there bars on two faces of the column or on all four faces? If the wrong curves are selected, the answers may be quite incorrect.

Although several methods are available for selecting column sizes, a trial-and-error method is about as good as any. With this procedure the designer estimates what he or she

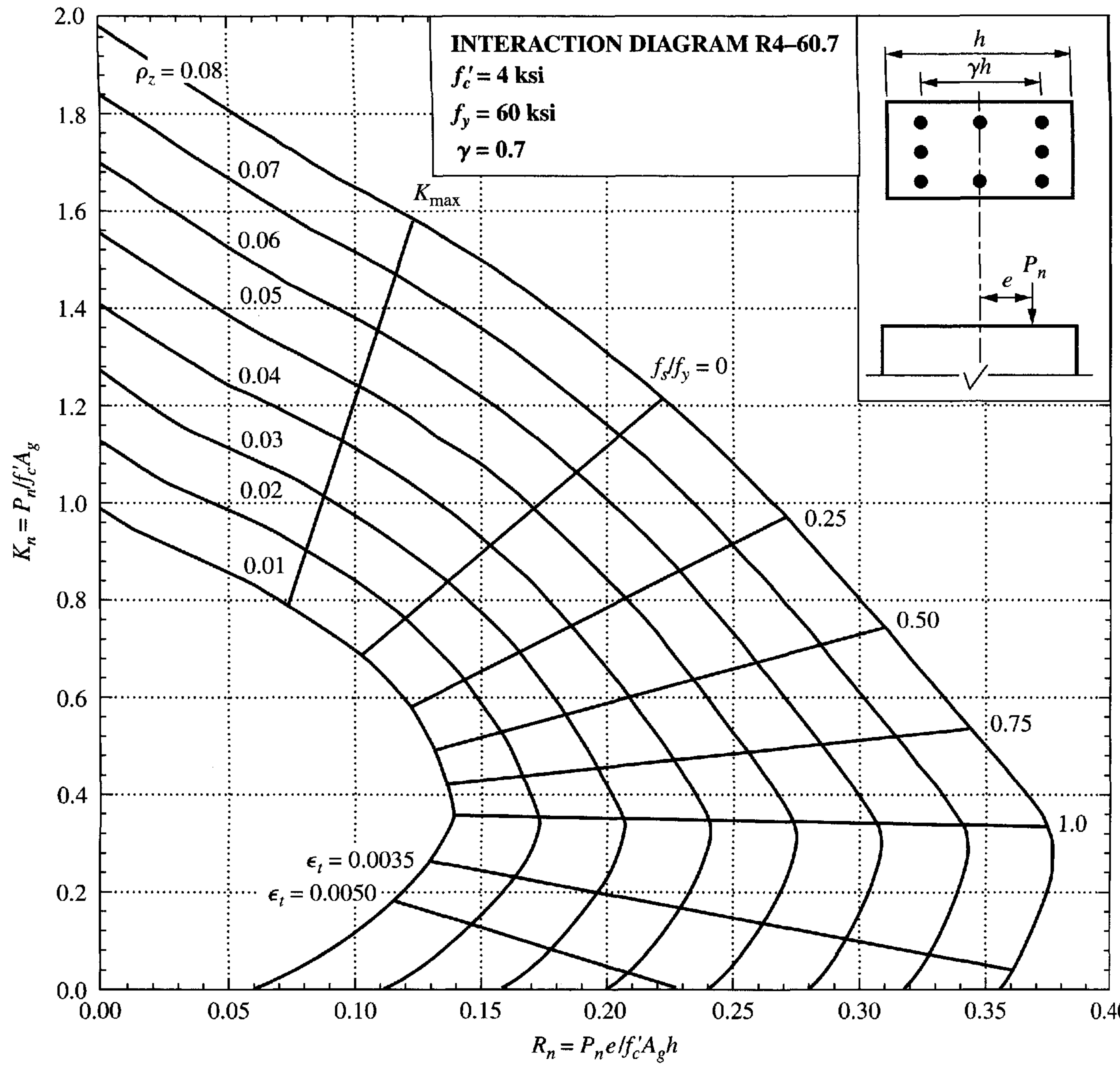

Figure 10.16 ACI rectangular column interaction diagram when bars are placed along all four faces. (Permission of American Concrete Institute.)

thinks is a reasonable column size and then determines the steel percentage required for that column size from the interaction diagram. If it is felt that the ρ determined is unreasonably large or small, another column size can be selected and the new required ρ selected from the diagrams, and so on. In this regard, the selection of columns for which ρ is greater than 4 or 5% results in congestion of the steel, particularly at splices, and consequent difficulties in getting the concrete down into the forms.

A slightly different approach is used in Example 10.4 where the average compression stress at ultimate load across the column cross section is assumed to equal some value—say, 0.5 to $0.6f'_c$. This value is divided into P_n to determine the column area required. Then cross-sectional dimensions are selected, and the value of ρ is determined from the interaction curves. Again, if the percentage obtained seems unreasonable, the column size can be revised and a new steel percentage obtained.

In Examples 10.3 to 10.5, reinforcing bars are selected for three columns. The values of $K_n = P_n / f'_c A_g$ and $R_n = P_n e / f'_c A_g h$ are computed. The position of those values is located on the appropriate graph, and ρ_g is determined and multiplied by the gross area of the column in question to determine the reinforcing area needed.

EXAMPLE 10.3

The short 14 × 20-in. tied column of Figure 10.17 is to be used to support the following loads and moments: $P_D = 125$ k, $P_L = 140$ k, $M_D = 75$ ft-k and $M_L = 90$ ft-k. If $f'_c = 4000$ psi and $f_y = 60{,}000$ psi, select reinforcing bars to be placed in its end faces only using appropriate ACI column interaction diagrams.

SOLUTION

$$P_u = (1.2)(125) + (1.6)(140) = 374 \text{ k}$$

$$P_n = \frac{374}{0.65} = 575.4 \text{ k}$$

$$M_u = (1.2)(75) + (1.6)(90) = 234 \text{ ft-k}$$

$$M_n = \frac{234}{0.65} = 360 \text{ ft-k}$$

$$e = \frac{(12)(360)}{575.4} = 7.51''$$

$$\gamma = \frac{15}{20} = 0.75$$

Compute values of K_n and R_n

$$K_n = \frac{P_n}{f'_c A_g} = \frac{575.4}{(4)(14 \times 20)} = 0.513$$

$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(575.4)(7.51)}{(4)(14 \times 20)(20)} = 0.193$$

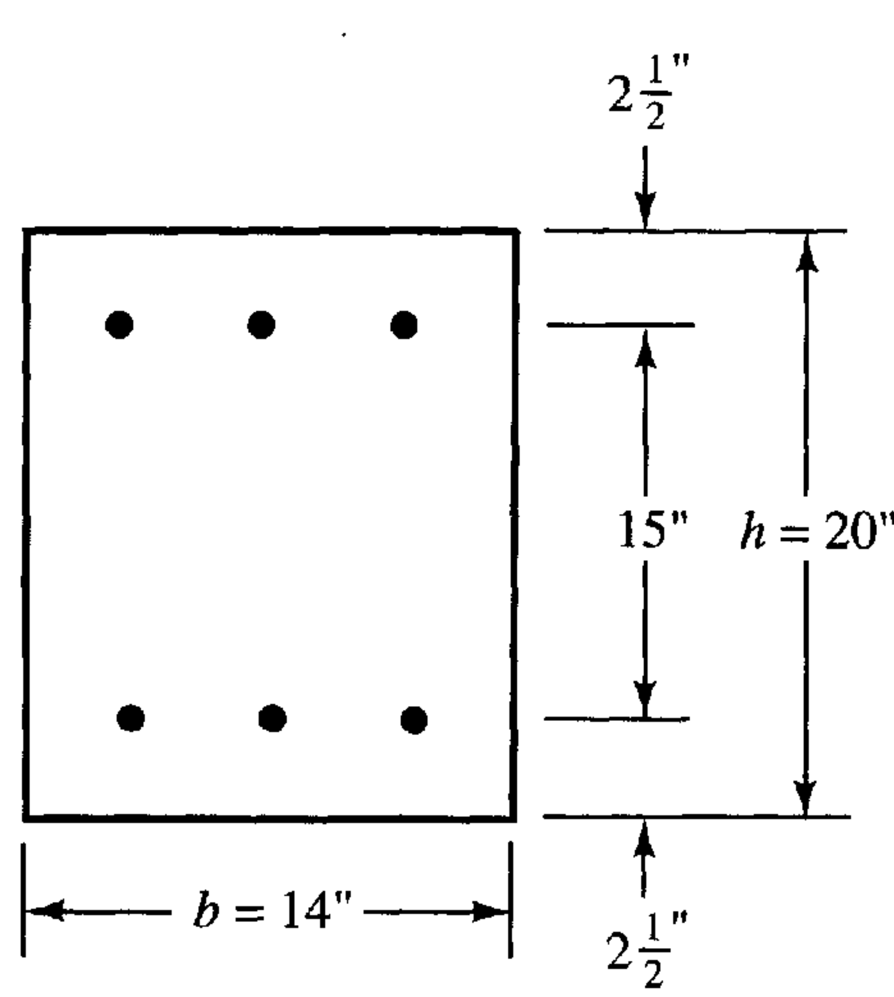

Figure 10.17

Value of γ falls between γ values for Graphs A.3 and A.4. Therefore interpolating between the two as follows:

γ	0.70	0.75	0.80
ρ_g	0.0220	0.0202	0.0185

$$A_s = \rho_g bh = (0.0202)(14)(20) = 5.66 \text{ in.}^2$$

<u>Use 6 #9 bars = 6.00 in.²</u>

Notes

(a) Note that $\phi = 0.65$ as initially assumed since the graphs used show $\frac{f_s}{f_y}$ is <1.0 and thus $\epsilon_t < 0.002$.

(b) Code requirements must be checked as in Example 9.1. (See Figure 10.25 to understand.)

EXAMPLE 10.4

Design a short square column for the following conditions: $P_u = 600$ k, $M_u = 125$ ft-k, $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi. Place the bars uniformly around all four faces of the column.

SOLUTION Assume the column will have an average compression stress = about $0.6f'_c = 2400$ psi.

$$A_g \text{ required} = \frac{600}{2.400} = 250 \text{ in.}^2$$

Try a 16 × 16-in. column ($A_g = 256$ in.^{2}) with the bar arrangement shown in Figure 10.18.

$$e = \frac{M_u}{P_u} = \frac{(12)(125)}{600} = 2.50''$$

$$P_n = \frac{P_u}{\phi} = \frac{600}{0.65} = 923.1 \text{ k}$$

$$K_n = \frac{P_n}{f'_c A_g} = \frac{923.1}{(4)(16 \times 16)} = 0.901$$

$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(600)(2.50)}{(4.0)(16 \times 16)(16)} = 0.0916$$

$$\gamma = \frac{11}{16} = 0.6875$$

Interpolating between values given in Graphs A.6 and A.7.

γ	0.600	0.6875	0.700
ρ_g	0.0240	0.0227	0.0225

$$A_s = (0.0227)(16)(16) = 5.81 \text{ in.}^2$$

<u>Use eight #8 bars = 6.28 in.²</u>

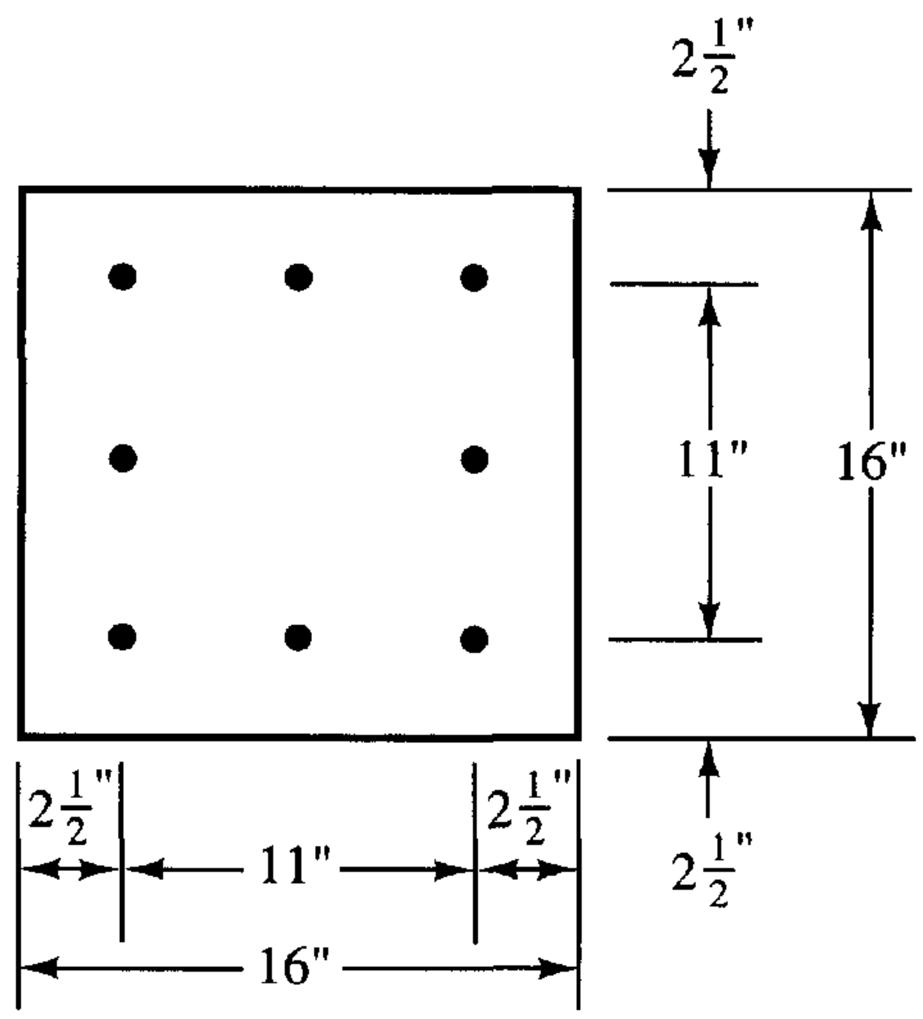

Figure 10.18

Notes

(a) Note that $\phi = 0.65$ as initially assumed since the graphs used show $\frac{f_s}{f_y} < 1.0$ and thus $\epsilon_t < 0.002$.

(b) Code requirements must be checked as in Example 9.1. (See Figure 10.25.)

EXAMPLE 10.5

Using the ACI column interaction graphs, select reinforcing for the short round spiral column shown in Figure 10.19 if $f'_c = 4000$ psi, $f_y = 60{,}000$ psi, $P_u = 500$ k, and $M_u = 200$ ft-k.

SOLUTION

$$e = \frac{(12)(200)}{500} = 4.80 \text{ in.}$$

$$P_n = \frac{P_u}{\phi} = \frac{500}{0.70} = 714.3 \text{ k}$$

$$K_n = \frac{P_n}{f'_c A_g} = \frac{714.3}{(4)(314)} = 0.569$$

$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(714.3)(4.80)}{(4)(314)(20)} = 0.136$$

$$\gamma = \frac{15}{20} = 0.75$$

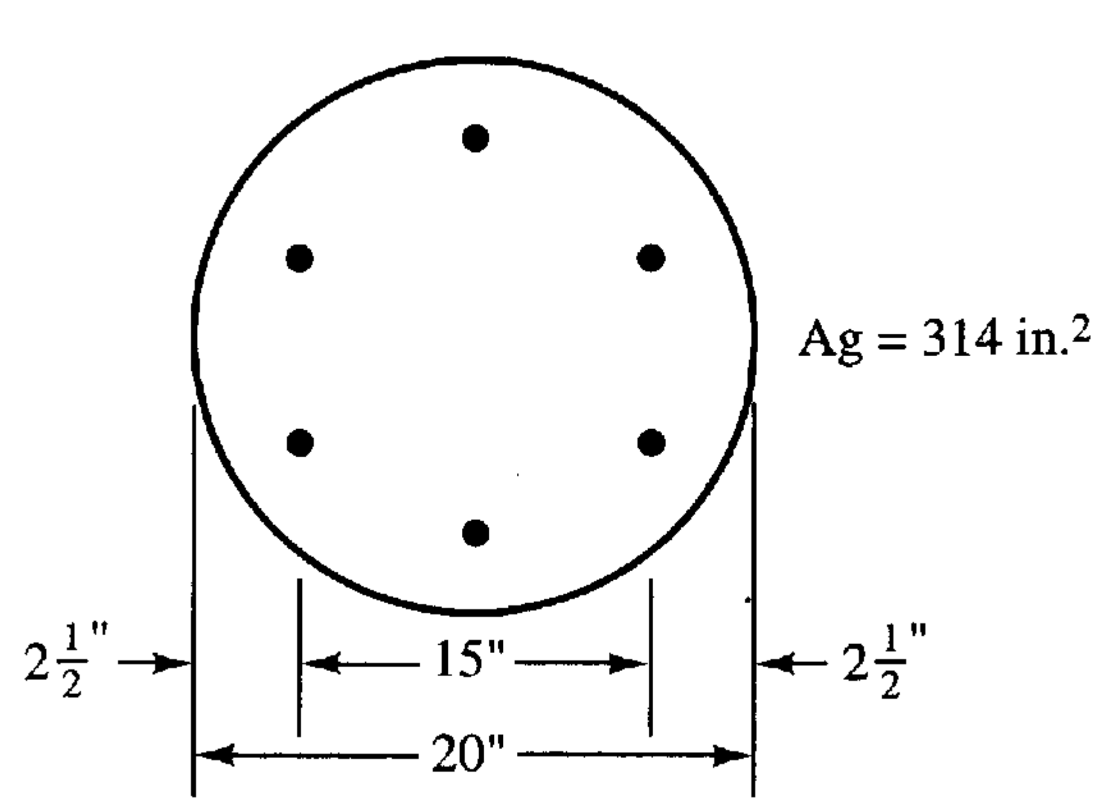

Figure 10.19

By interpolation between Graphs A.11 and A.12 ρ_g is found to equal 0.0235 and $\frac{f_s}{f_y} < 1.0$.

$$\rho A_g = (0.0235)(314) = 7.38 \text{ in.}^2$$

Use 8 #9 bars = 8.00 in.^{2}

Same notes as for Examples 10.3 and 10.4.

In Example 10.6 it is desired to select a 14-in. wide column with approximately 2% steel. This is done by trying different column depths and then determining the steel percentage required in each case.

EXAMPLE 10.6

Design a 14-in. wide rectangular short tied column with bars only in the two end faces for $P_u = 500$ k, $M_u = 250$ ft-k, $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi. Select a column with approximately 2% steel.

SOLUTION

$$e = \frac{M_u}{P_u} = \frac{(12)(250)}{500} = 6.00''$$

$$P_n = \frac{P_u}{\phi} = \frac{500}{0.65} = 769.2 \text{ k}$$

Trying several column sizes (14 × 20, 14 × 22, 14 × 24) and determining reinforcing.

Trial sizes	14 × 20	14 × 22	14 × 24
$K_n = \dfrac{P_n}{f'_c A_g}$	0.687	0.624	0.572
$R_n = \dfrac{P_n e}{f'_c A_g h}$	0.206	0.170	0.143
$\gamma = \dfrac{h - 2 \times 2.50}{h}$	0.750	0.773	0.792
ρ_g by interpolation	0.0315	0.0202	0.0109

Use 14 × 22 column

$$A_g = (0.0202)(14 \times 22) = 6.22 \text{ in.}^2$$

Use 8 #8 bars = 6.28 in.^{2}

Same notes as for Examples 10.3 and 10.4.

One more illustration of the use of the ACI interaction diagrams is presented with Example 10.7. In this example, the nominal column load P_n at a given eccentricity which a column can support is determined.

With reference to the ACI interaction curves, the reader should carefully note that the value of R_n (which is $P_n e / f'_c A_g h$) for a particular column, equals e/h times the value of K_n ($P_n / f'_c A_g$) for that column. This fact needs to be understood when the user desires to determine the nominal load that a column can support at a given eccentricity.

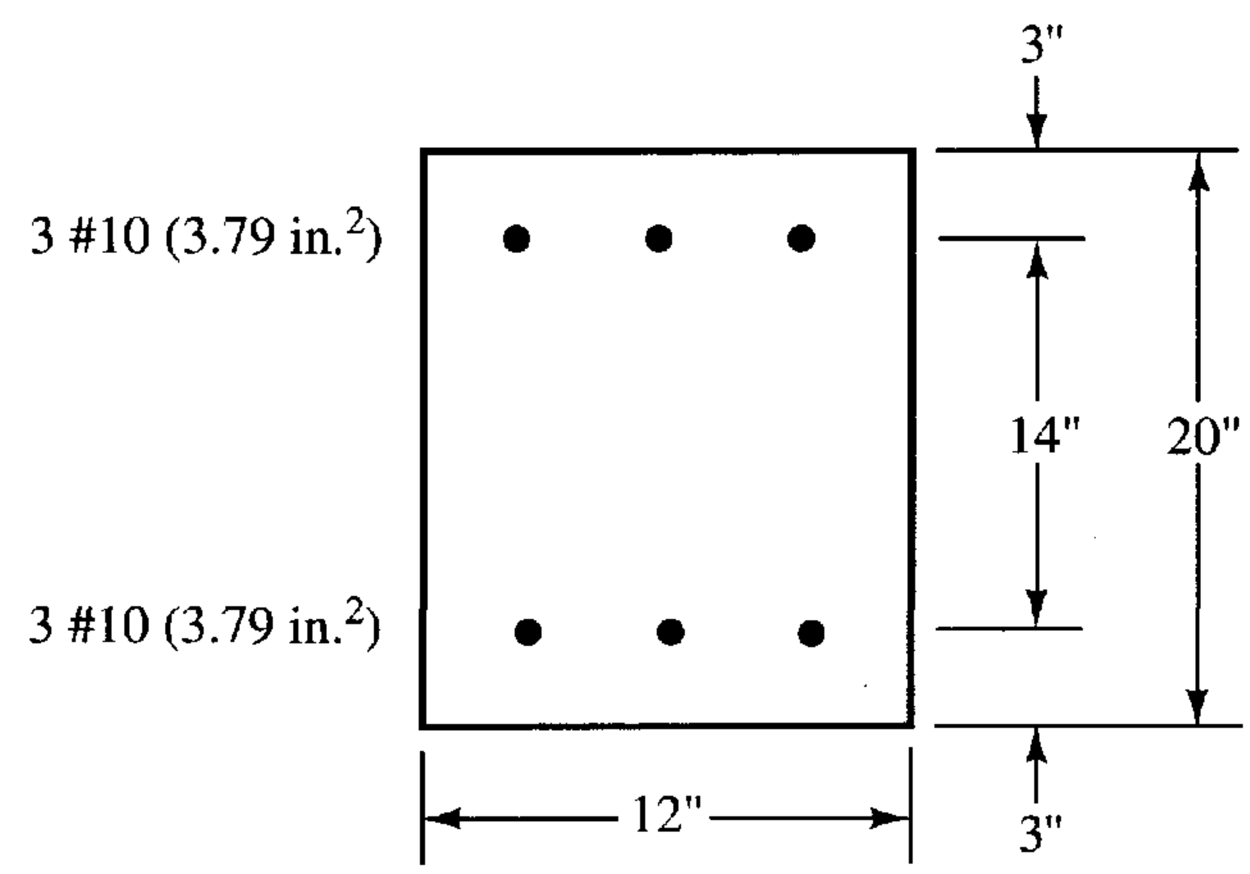

Figure 10.20

In Example 10.7 the nominal load that the short column of Figure 10.20 can support at an eccentricity of 10 in. with respect to the x axis is determined. If we plot on the interaction diagram the intersection point of K_n and R_n for a particular column and draw a straight line from that point to the lower left corner or origin of the figure, we will have a line with a constant e/h. For the column of Example 10.6 $e/h = 10/20 = 0.5$. Therefore a line is plotted from the origin through a set of assumed values for K_n and R_n in the proportion of 10/20 to each other. In this case, K_n was set equal to 0.8 and $R_n = 0.5 \times 0.8 = 0.4$. Next a line was drawn from that intersection point to the origin of the diagram as shown in Figure 10.16. Finally, the intersection of this line with ρ_g (0.0316 in this example) was determined, and the value of K_n or R_n was read. This latter value enables us to compute P_n.

EXAMPLE 10.7

Using the appropriate interaction curves, determine the value of P_n for the short tied column shown in Figure 10.20 if $e_x = 10''$. Assume $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

SOLUTION

$$\frac{e}{h} = \frac{10}{20} = 0.500$$

$$\rho_g = \frac{(2)(3.79)}{(12)(20)} = 0.0316$$

$$\gamma = \frac{14}{20} = 0.700$$

Plotting a straight line through the origin and the intersection of assumed values of K_n and R_n (say 0.8 and 0.4, respectively).

For ρ_g of 0.0316 we read the value of R_n

$$R_n = \frac{P_n e}{f'_c A_g h} = 0.24$$

$$P_n = \frac{(0.24)(4)(12 \times 20)(20)}{10} = 460.8\text{k}$$

When the usual column is subjected to axial load and moment, it seems reasonable to assume initially that $\phi = 0.65$ for tied columns and 0.70 for spiral columns. It is to be