Method of Least Work

Theory of Structures-II

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Method of Least Work / Castigliano's Second Theorem

Statically Indeterminate Trusses

- Strain energy stored or work done in a member of a truss is

$$U = \frac{P^2 L}{2AE}$$

- In the entire truss, the strain energy is

$$U = \sum \frac{P^2 L}{2AE}$$

where *P* is the axial force in the member.

Statically Indeterminate Trusses

- Differential co-efficient of *U* with respect to a force *S* in the member is

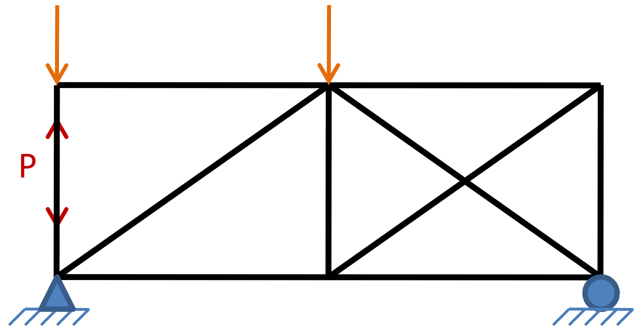
$$\frac{\partial U}{\partial S} = \sum \frac{PL}{AE}\frac{\partial P}{\partial S}$$

- According to the Theorem of Least Work

$$\frac{\partial U}{\partial S} = 0$$

$$\frac{\partial U}{\partial S} = \sum \frac{PL}{AE}\frac{\partial P}{\partial S} = 0$$

Statically Indeterminate Trusses



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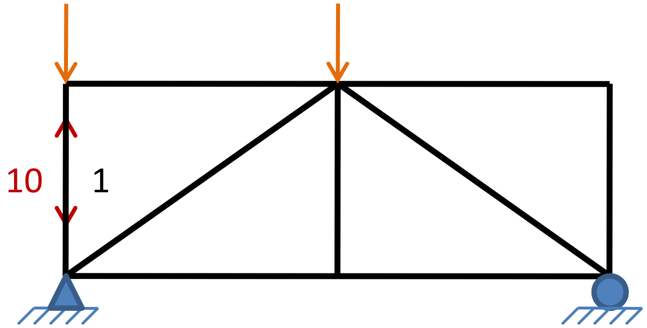

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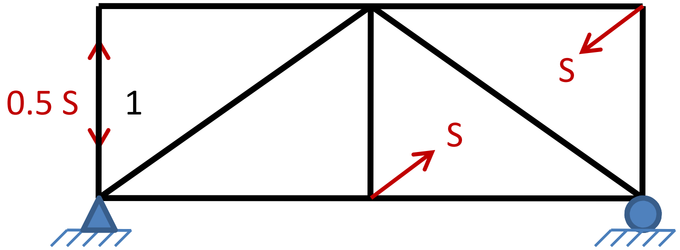
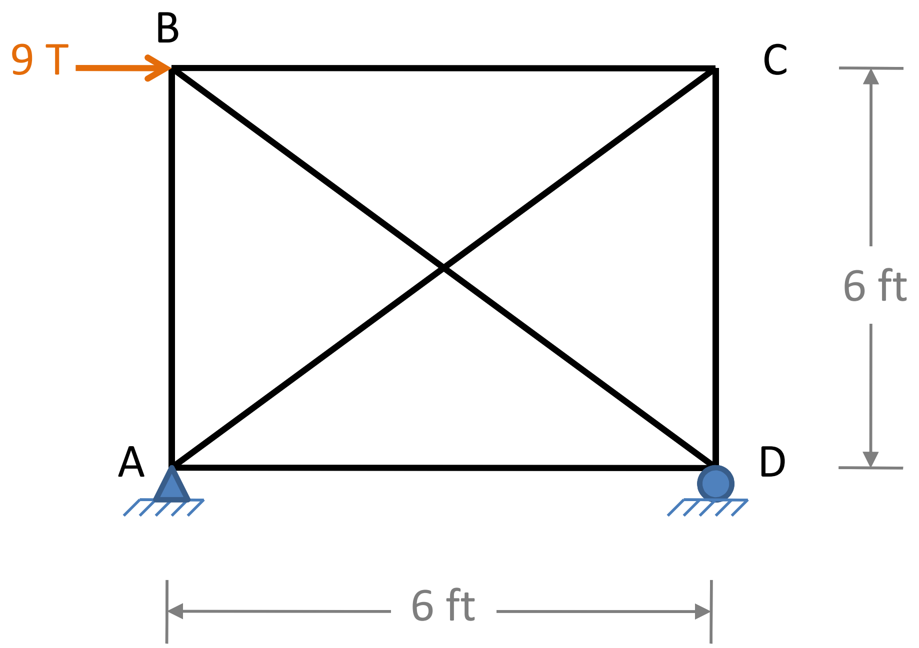


$$P = 10 + 0.5S$$

$$\frac{\partial U}{\partial S} = \sum \frac{PL}{AE}\frac{\partial P}{\partial S} = 0$$

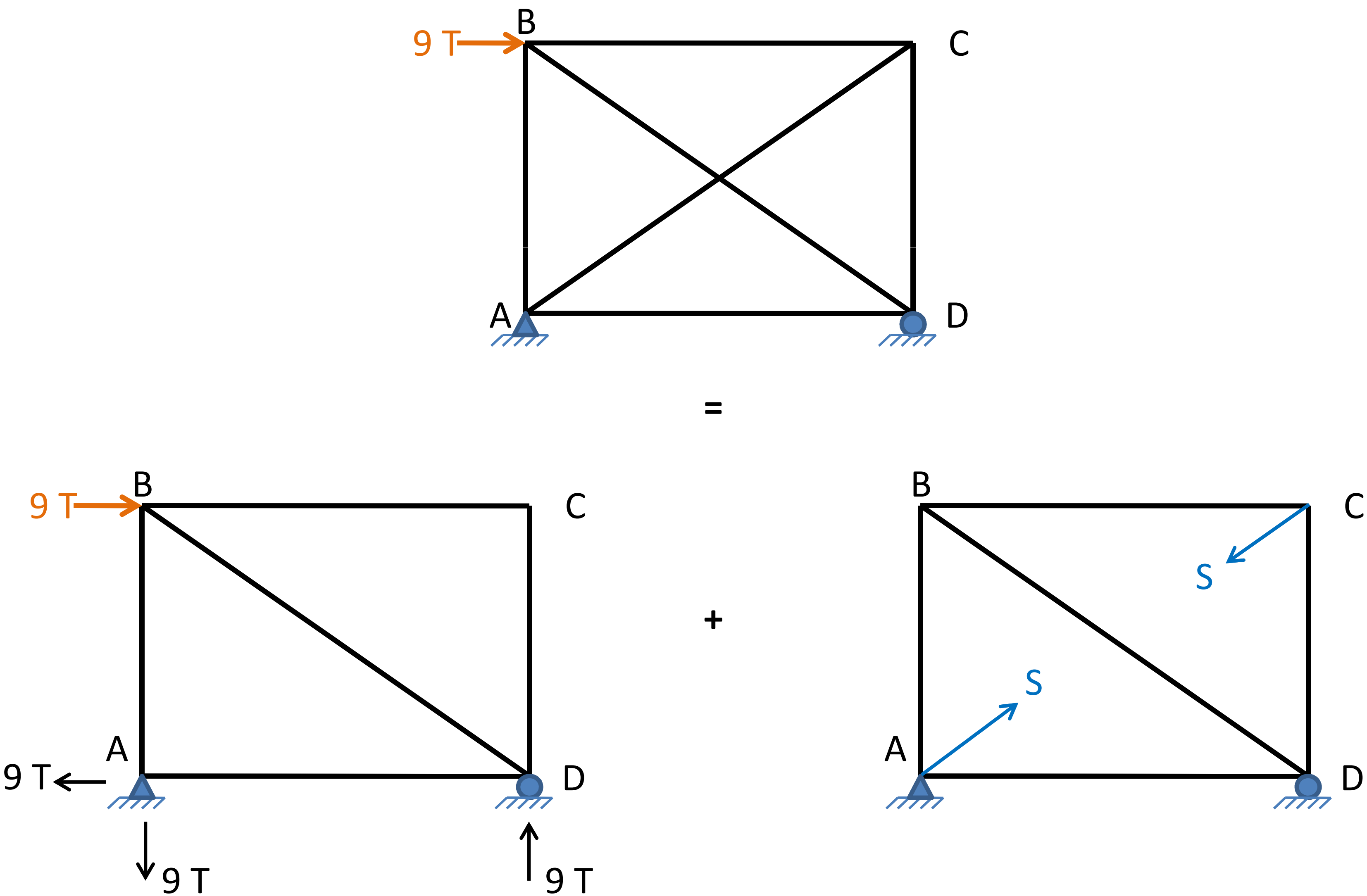
Example 1

Determine the force in members of the truss shown. AE is same for all members.

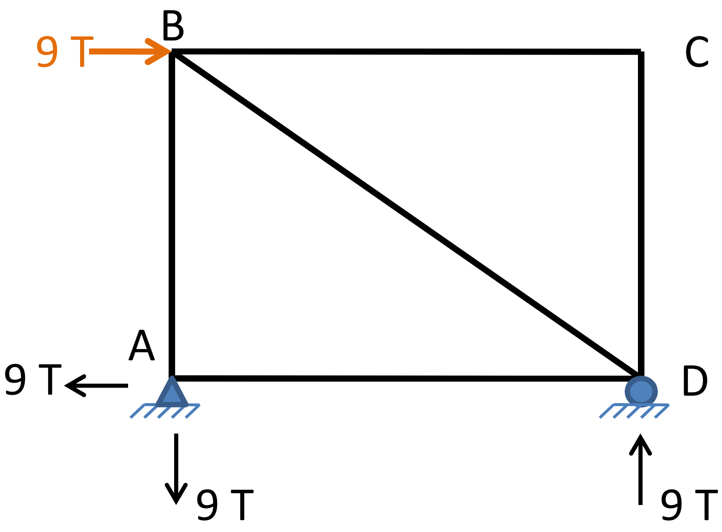


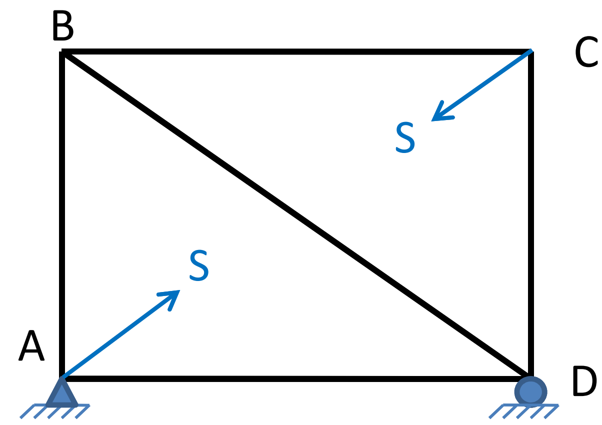
Solution

The truss is indeterminate to the first degree.



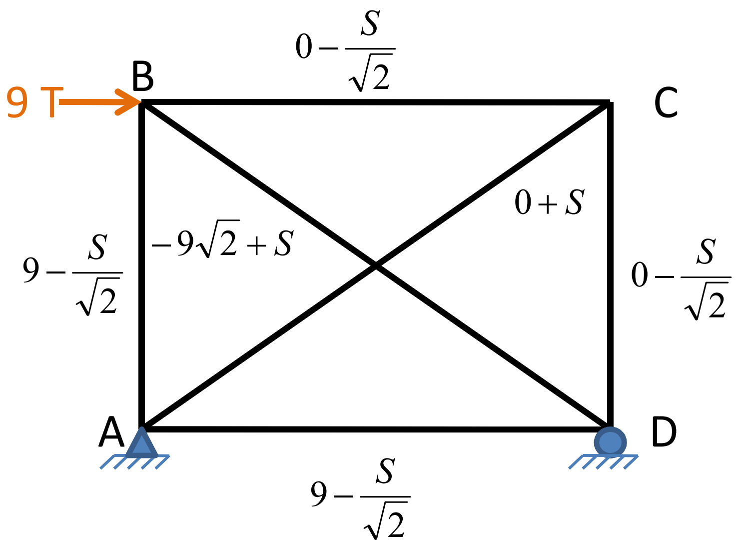
Solution





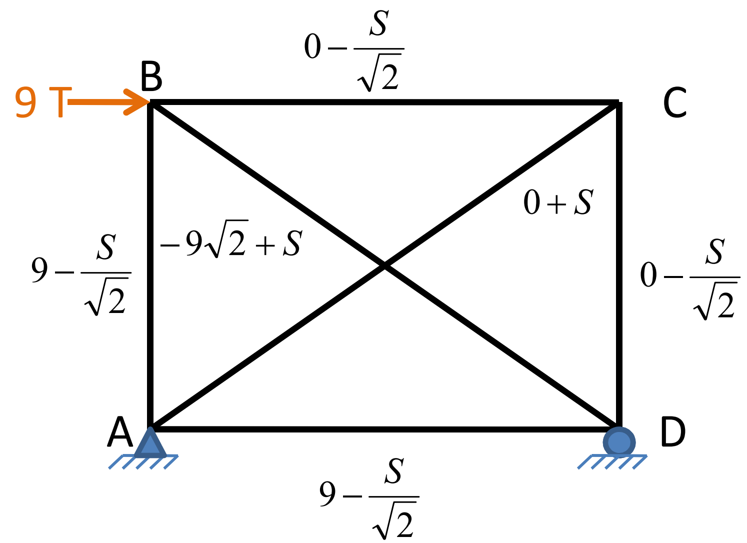
Member	Force due to Applied Load	Force due to Redundant Member Force	P
AB	9	$-S/\sqrt{2}$	$9 - S/\sqrt{2}$
BC	0	$-S/\sqrt{2}$	$0 - S/\sqrt{2}$
CD	0	$-S/\sqrt{2}$	$0 - S/\sqrt{2}$
AD	9	$-S/\sqrt{2}$	$9 - S/\sqrt{2}$
BD	$-9\sqrt{2}$	+S	$-9\sqrt{2} + S$
AC	0	+S	S

Solution



Member	Force due to Applied Load	Force due to Redundant Member Force	P
AB	9	$-S/\sqrt{2}$	$9 - S/\sqrt{2}$
BC	0	$-S/\sqrt{2}$	$0 - S/\sqrt{2}$
CD	0	$-S/\sqrt{2}$	$0 - S/\sqrt{2}$
AD	9	$-S/\sqrt{2}$	$9 - S/\sqrt{2}$
BD	$-9\sqrt{2}$	+S	$-9\sqrt{2} + S$
AC	0	+S	S

Solution



Member	P	∂P/∂S	L (in.)	PL	PL∂P/∂S
AB	$9-S/\sqrt{2}$	$-1/\sqrt{2}$	72	$648-\frac{72}{\sqrt{2}}S$	$\frac{-648}{\sqrt{2}}+36S$
BC	$0-S/\sqrt{2}$	$-1/\sqrt{2}$	72	$-\frac{72}{\sqrt{2}}S$	$+36S$
CD	$0-S/\sqrt{2}$	$-1/\sqrt{2}$	72	$-\frac{72}{\sqrt{2}}S$	$+36S$
AD	$9-S/\sqrt{2}$	$-1/\sqrt{2}$	72	$648-\frac{72}{\sqrt{2}}S$	$\frac{-648}{\sqrt{2}}+36S$
BD	$-9\sqrt{2}+S$	$+1$	$72\sqrt{2}$	$-1296+72\sqrt{2}S$	$-1296+101.8S$
AC	S	$+1$	$72\sqrt{2}$	$72\sqrt{2}S$	$101.8S$

$$\sum PL\frac{\partial P}{\partial S}=-2214.4+347.6S$$

Solution

$$\frac{1}{AE}\sum PL\frac{\partial P}{\partial S}=0$$

$$\sum PL\frac{\partial P}{\partial S}=0$$

$$-2214.4+347.6S=0$$

$$S=6.38\,T$$

Substituting value of S in 2nd Column

$$P_{AB}=4.49\,T$$

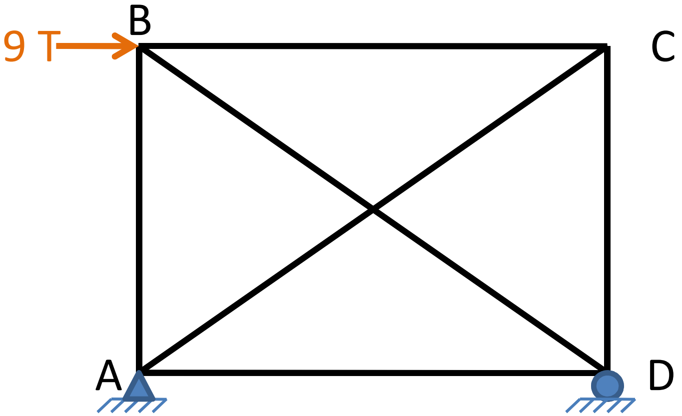
$$P_{BC}=-4.51\,T$$

$$P_{CD}=-4.51\,T$$

$$P_{AD}=4.49\,T$$

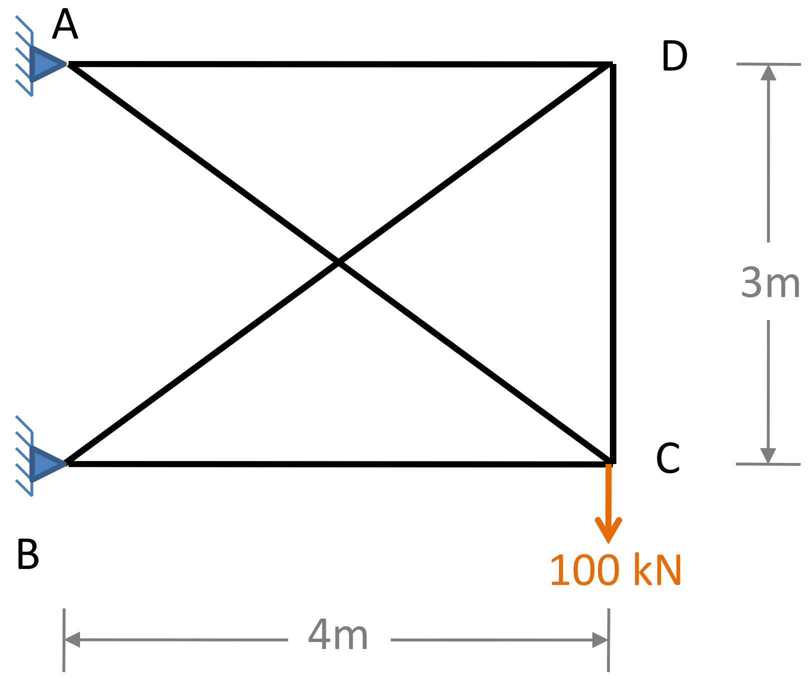
$$P_{BD}=-6.35\,T$$

$$P_{AC}=6.38\,T$$



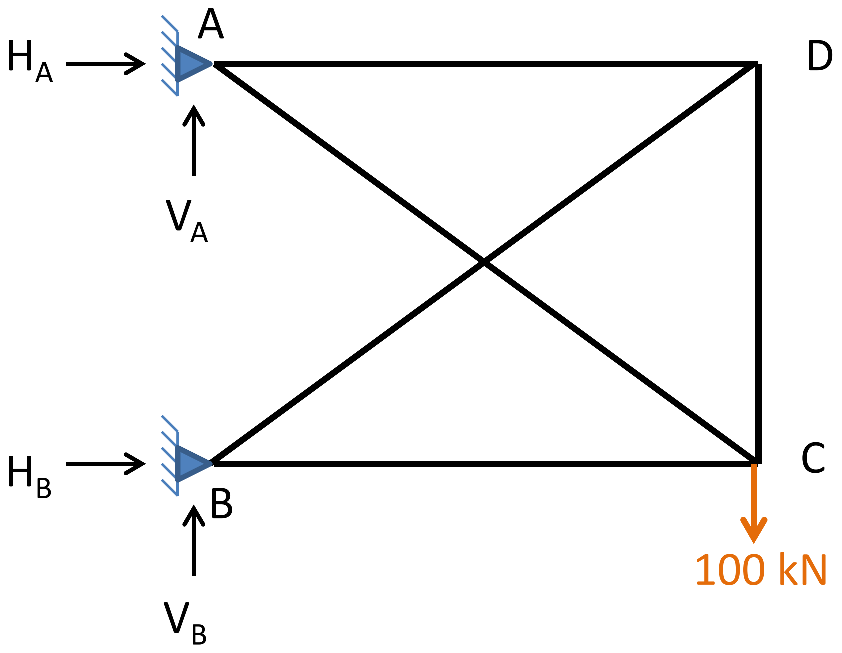
Example 2

Analyze the truss shown. AE is same for all members.

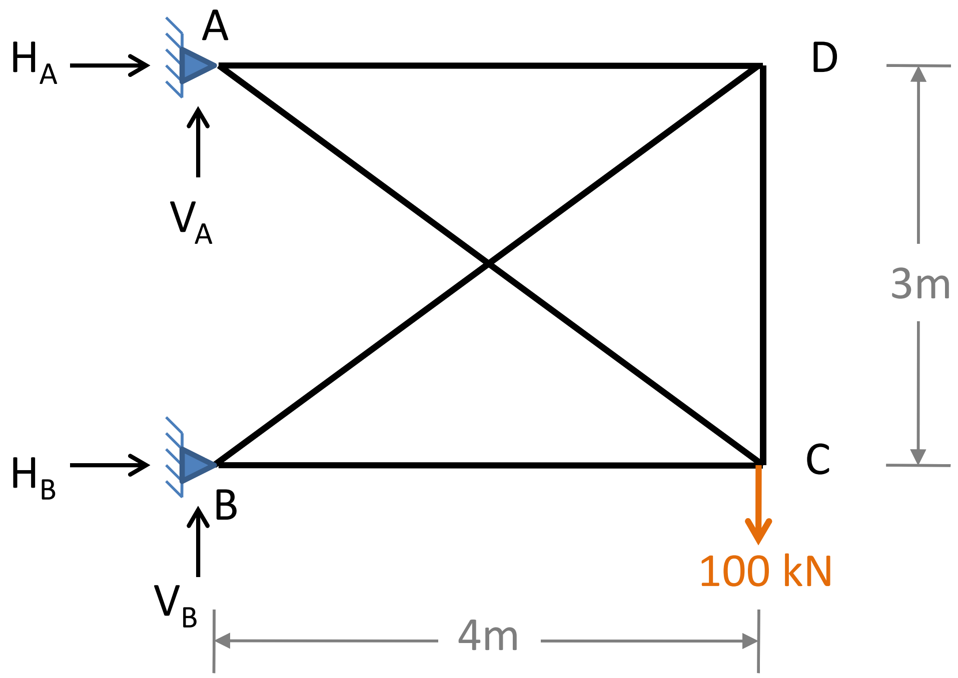


Solution

- The truss is externally indeterminate to the first degree.
- Let V_B be the redundant.
- Find the other reactions in terms of redundant V_B.



Solution



$$\sum M_A = 0$$

$$-100 \times 4 + H_B \times 3 = 0$$

$$-400 + 3H_B = 0$$

$$H_B = \frac{400}{3} = 133.33\, kN$$

$$\sum F_x = 0$$

$$H_A + H_B = 0$$

$$H_A = -H_B = -133.33$$

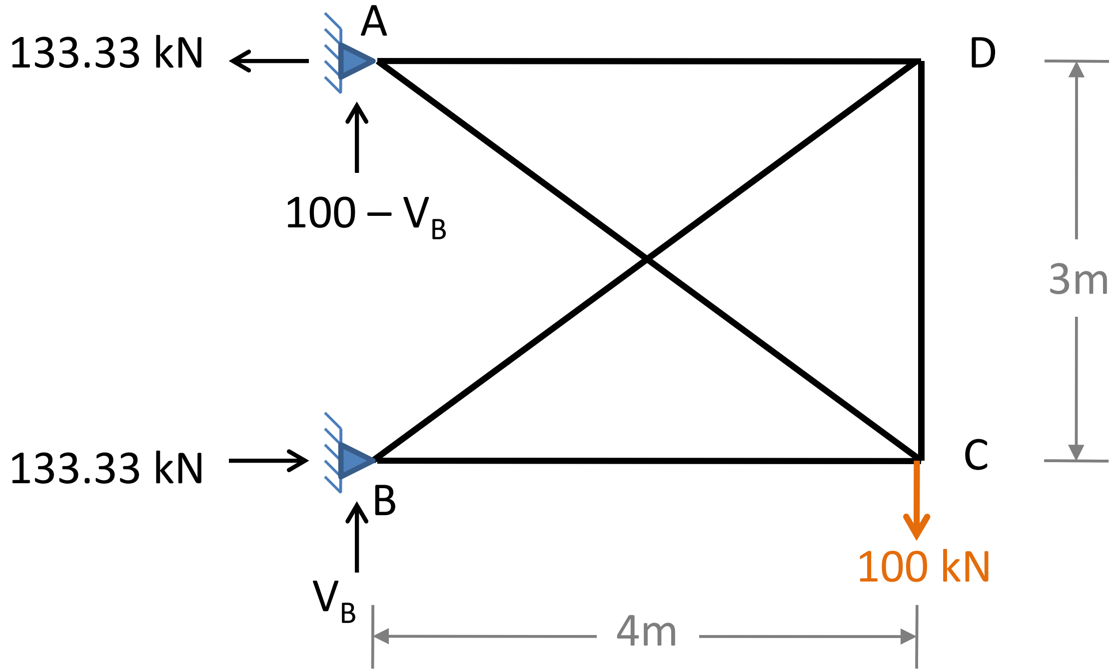
$$H_A = -133.33\, kN$$

$$\sum F_y = 0$$

$$V_A + V_B = 100$$

$$V_A = 100 - V_B$$

Solution



$$\sum M_A = 0$$
$$-100 \times 4 + H_B \times 3 = 0$$
$$-400 + 3H_B = 0$$
$$H_B = \frac{400}{3} = 133.33\,kN$$

$$\sum F_x = 0$$
$$H_A + H_B = 0$$
$$H_A = -H_B = -133.33$$
$$H_A = -133.33\,kN$$

$$\sum F_y = 0$$
$$V_A + V_B = 100$$
$$V_A = 100 - V_B$$

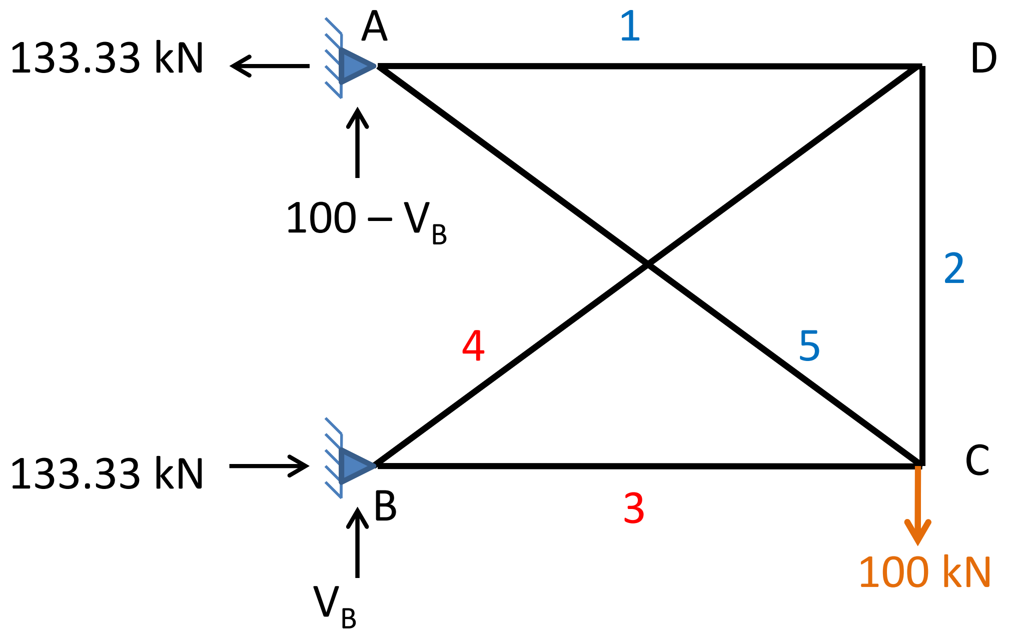
Solution

Joint B

$$\sum F_y = 0$$

$$V_B + F_4 \times \frac{3}{5} = 0$$

$$F_4 = -V_B \times \frac{5}{3}$$

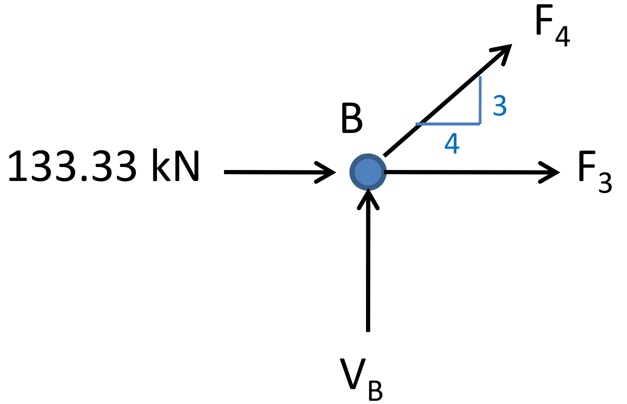


$$\sum F_x = 0$$

$$133.33 + F_3 + F_4 \times \frac{4}{5} = 0$$

$$F_3 = -133.33 + V_B \times \frac{5}{3} \times \frac{4}{5}$$

$$F_3 = -133.33 + 1.33 V_B$$



Solution

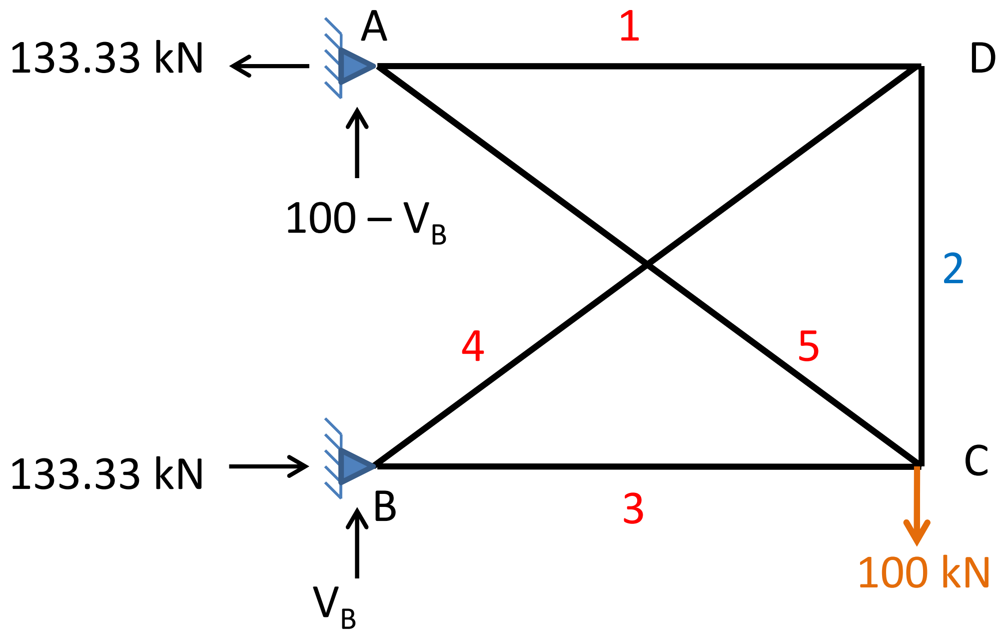
Joint A

$$\sum F_y = 0$$

$$100 - V_B - F_5 \times \frac{3}{5} = 0$$

$$F_5 \times \frac{3}{5} = 100 - V_B$$

$$F_5 = \frac{5}{3} \times 100 - \frac{5}{3} V_B$$



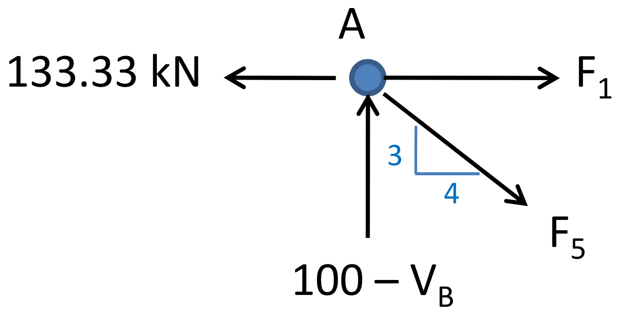
$$\sum F_x = 0$$

$$F_1 - 133.33 + F_5 \times \frac{4}{5} = 0$$

$$F_1 = 133.33 - F_5 \times \frac{4}{5} = 133.33 - \frac{4}{5}\left(\frac{5}{3} \times 100 - \frac{5}{3} V_B\right)$$

$$F_1 = 133.33 - 133.33 + 1.333 V_B$$

$$F_1 = +1.333 V_B$$



Solution

Joint D

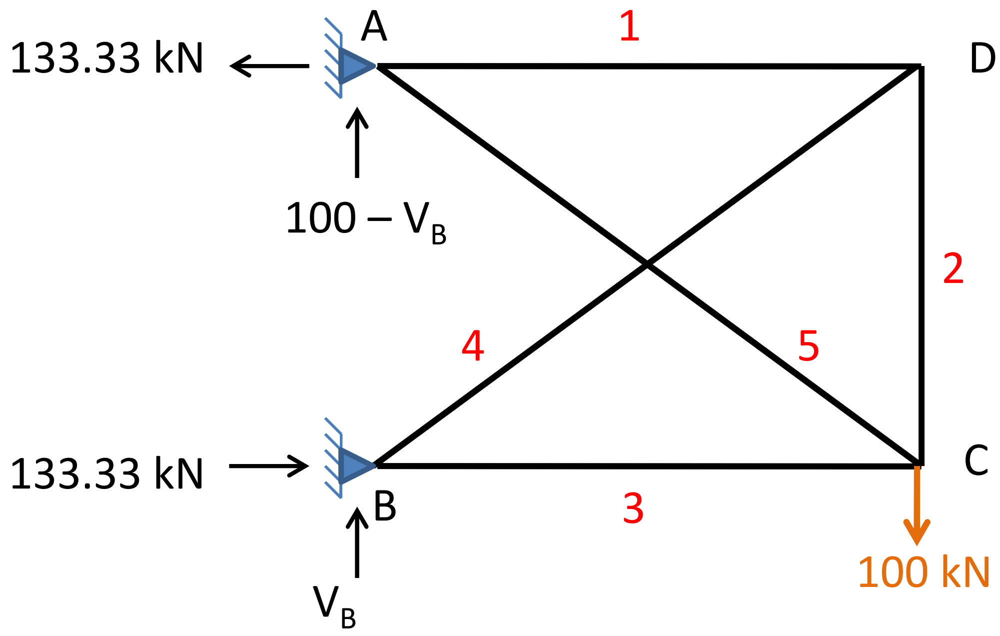
$$\sum F_y = 0$$

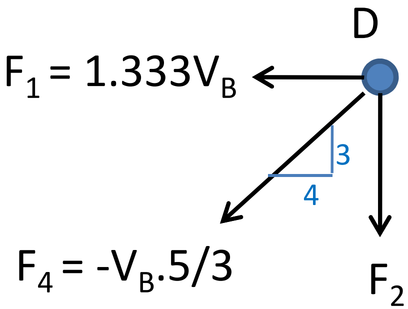
$$F_2 + F_4 \frac{3}{5} = 0$$

$$F_2 = -F_4 \times \frac{3}{5}$$

$$F_2 = +V_B \times \frac{5}{3} \times \frac{3}{5}$$

$$F_2 = +V_B$$





Solution

The strain energy of a truss is

$$U = \sum \frac{F^2 L}{2AE}$$

$$\frac{\partial U}{\partial V_B} = \sum \frac{\partial F}{\partial V_B} \frac{FL}{AE}$$

See Table on next slide

Solution

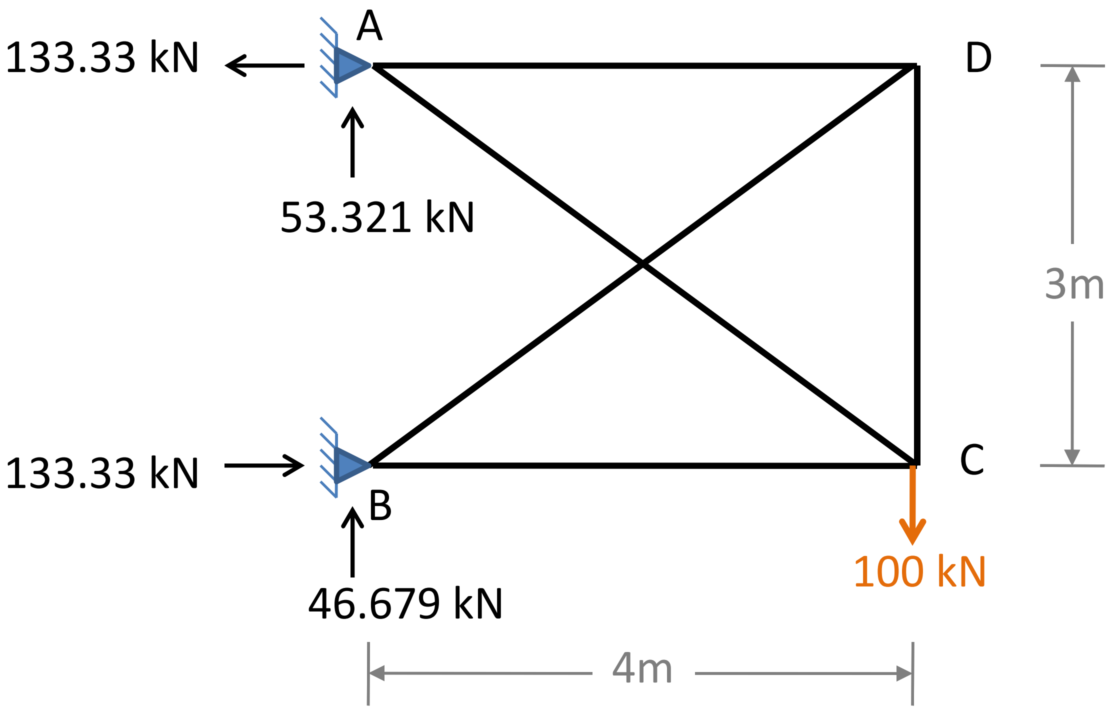
Member	F	$\partial F/\partial V_B$	L (m)	$FL.\ \partial F/\partial V_B$	Final Forces
1	$+1.33V_B$	+1.333	4	$7.1075V_B$	+62.223 kN
2	$+V_B$	+1	3	$3V_B$	+46.679 kN
3	$-133.33 +1.33V_B$	+1.333	4	$-709.315+7.075V_B$	-71.246 kN
4	$-V_B \times 5/3$	-1.667	5	$+13.894V_B$	-77.798 kN
5	$5/3(100-V_B)$	-1.667	5	$-1389.44+13.894V_B$	+88.868 kN

$$\sum \frac{\partial F}{\partial V_B} FL = 0$$

$$(7.1075 + 3 + 7.075 + 13.894 + 13.894)V_B - (709.315 + 1389.44) = 0$$

$$V_B = 46.679\ kN$$

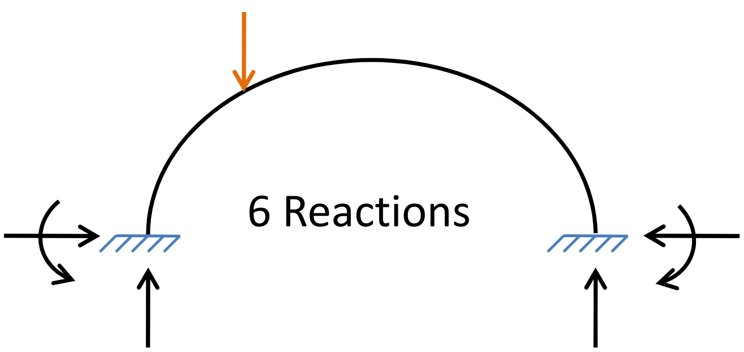
Solution



Method of Least Work

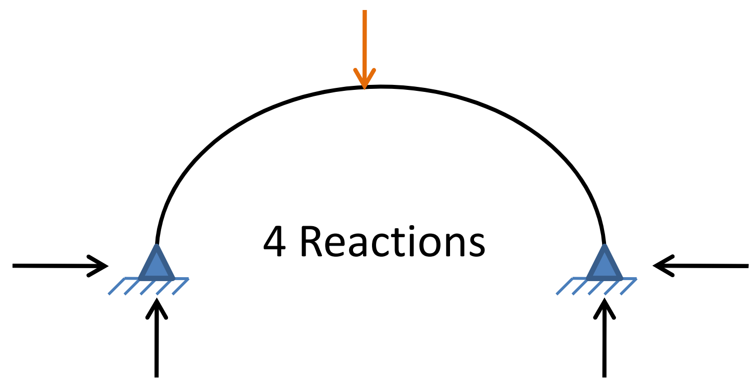
Statically Indeterminate Arches

Fixed Arches



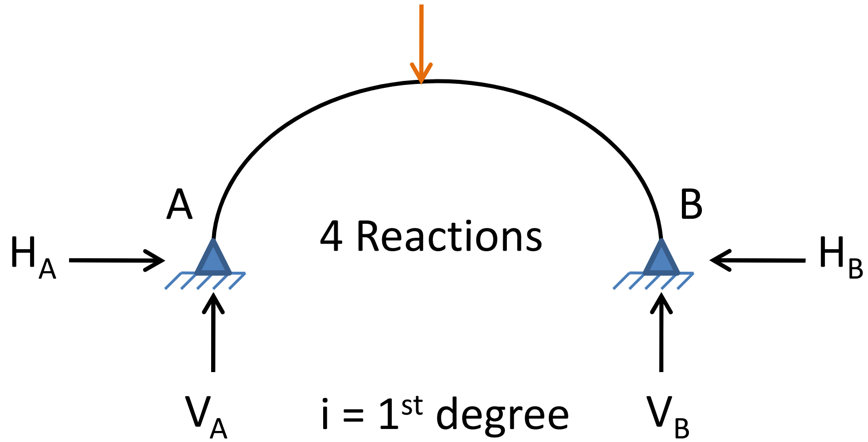
i = 3rd degree

Two Hinged Arches



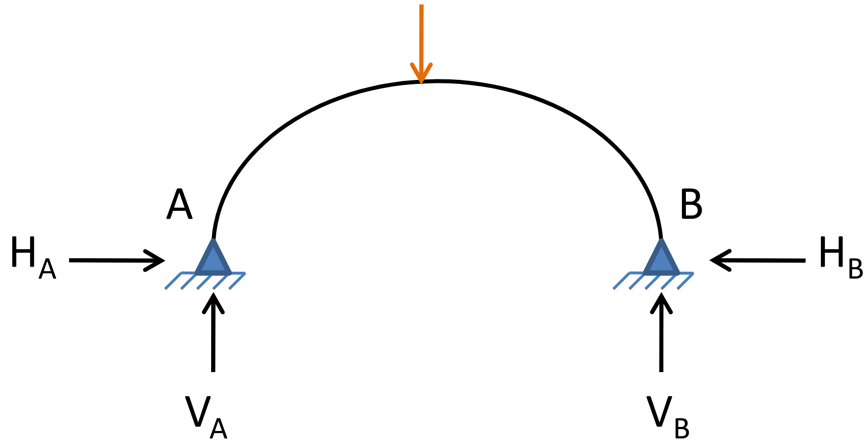
i = 1st degree

Two Hinged Arch



- We have 4 unknowns, V_A, H_A, V_B, and H_B but only three equations of equilibrium.
- $\sum H = 0$, $H_A = H_B = H$, when no external horizontal force is acting on the arch.
- If the supports are unyielding $\partial U / \partial H = 0$, which gives the fourth equation for solving it.

Two Hinged Arch

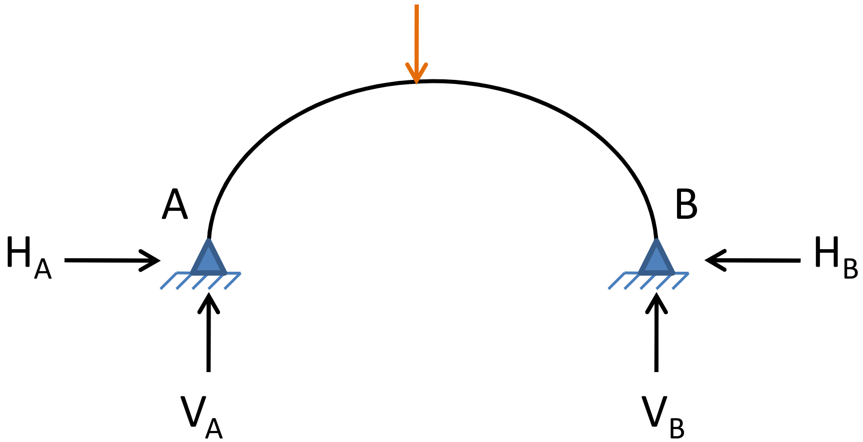


- An arch is subjected to BM, SF and Axial Thrust N.

- Total strain energy for the arch is given by

$$U = \int_0^L \frac{M^2}{2EI} ds + \int_0^L \frac{V^2}{2GA_s} ds + \int_0^L \frac{F^2}{2GAE} ds$$

Two Hinged Arch



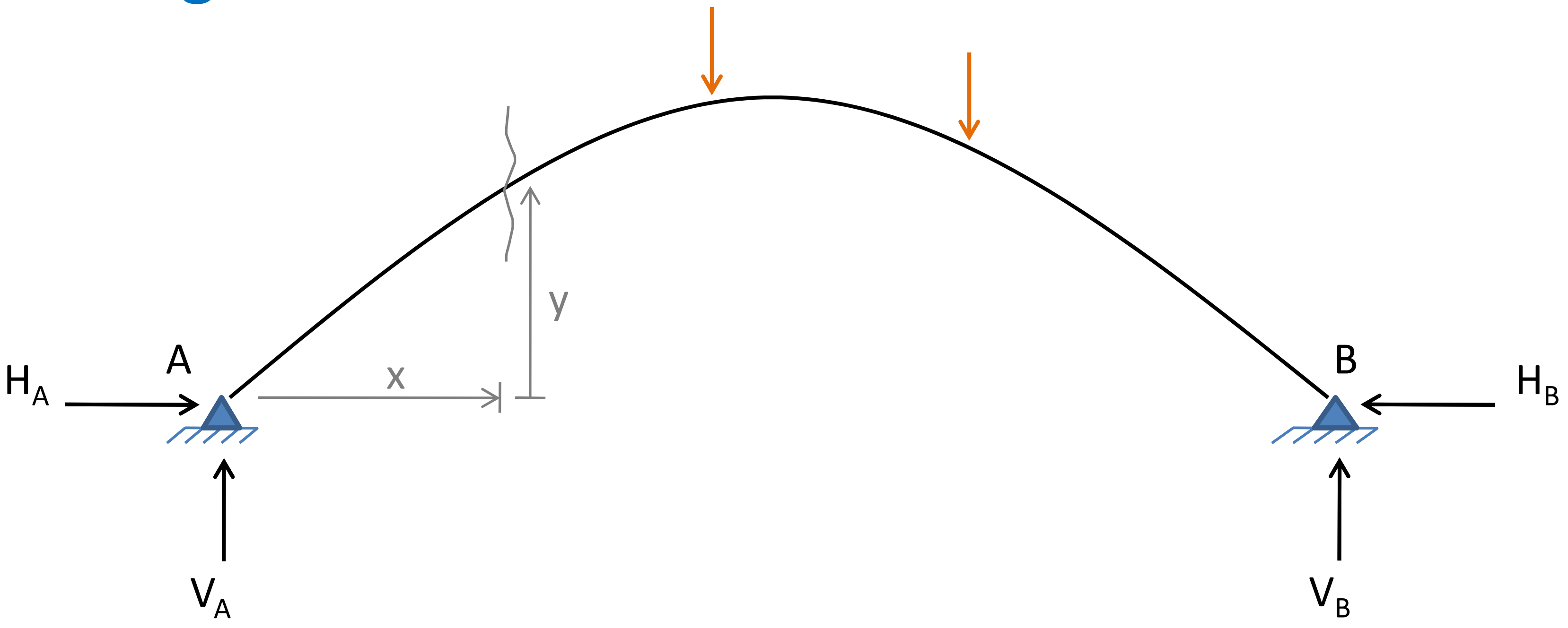
- If we consider only bending deformation for simplified analysis, then

$$U = \int_0^L \frac{M^2}{2EI} ds$$

- According to Theorem of Least Work

$$\frac{\partial U}{\partial H} = \int_0^L \frac{\partial M}{\partial H} \frac{M}{EI} ds \qquad (1)$$

Two Hinged Parabolic Arch



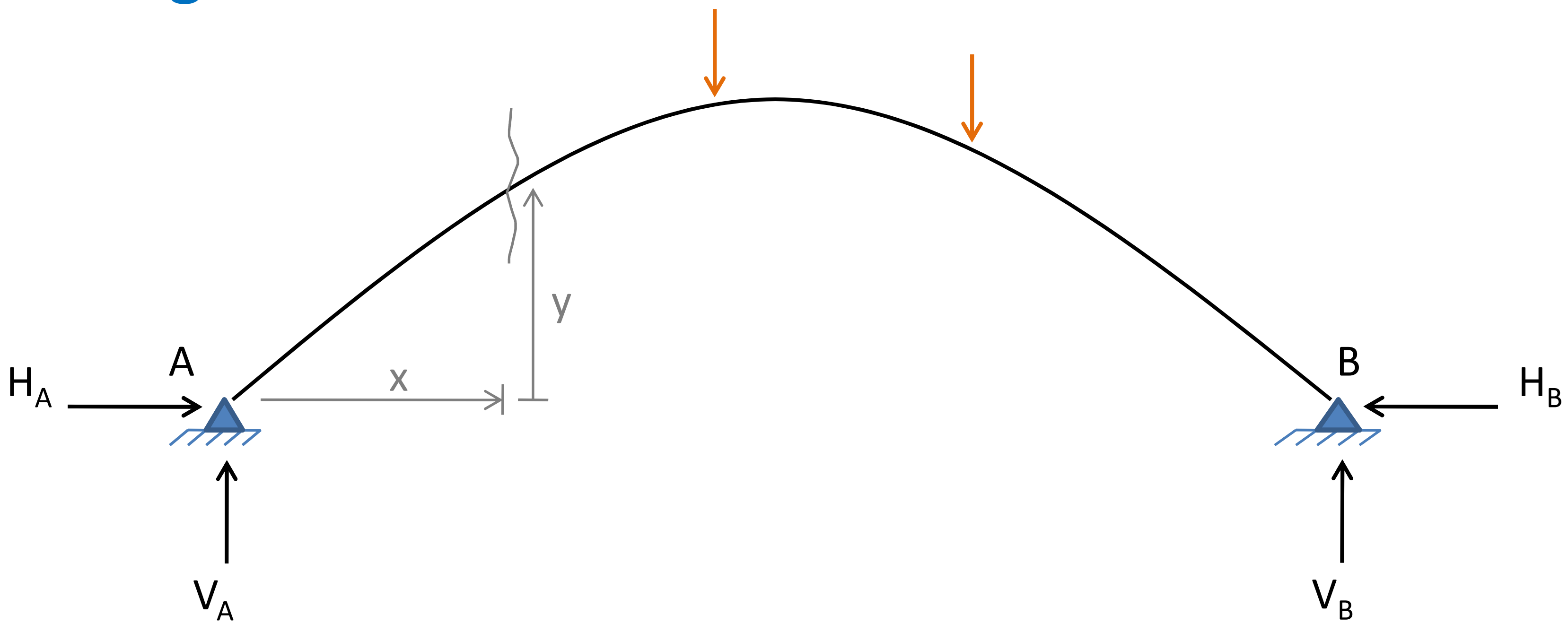
- For an Arch the bending moment is

$$M = V_A.x - H_A.y$$

- if $V_A.x = \mu$

$$M = \mu - H_A.y$$

Two Hinged Parabolic Arch



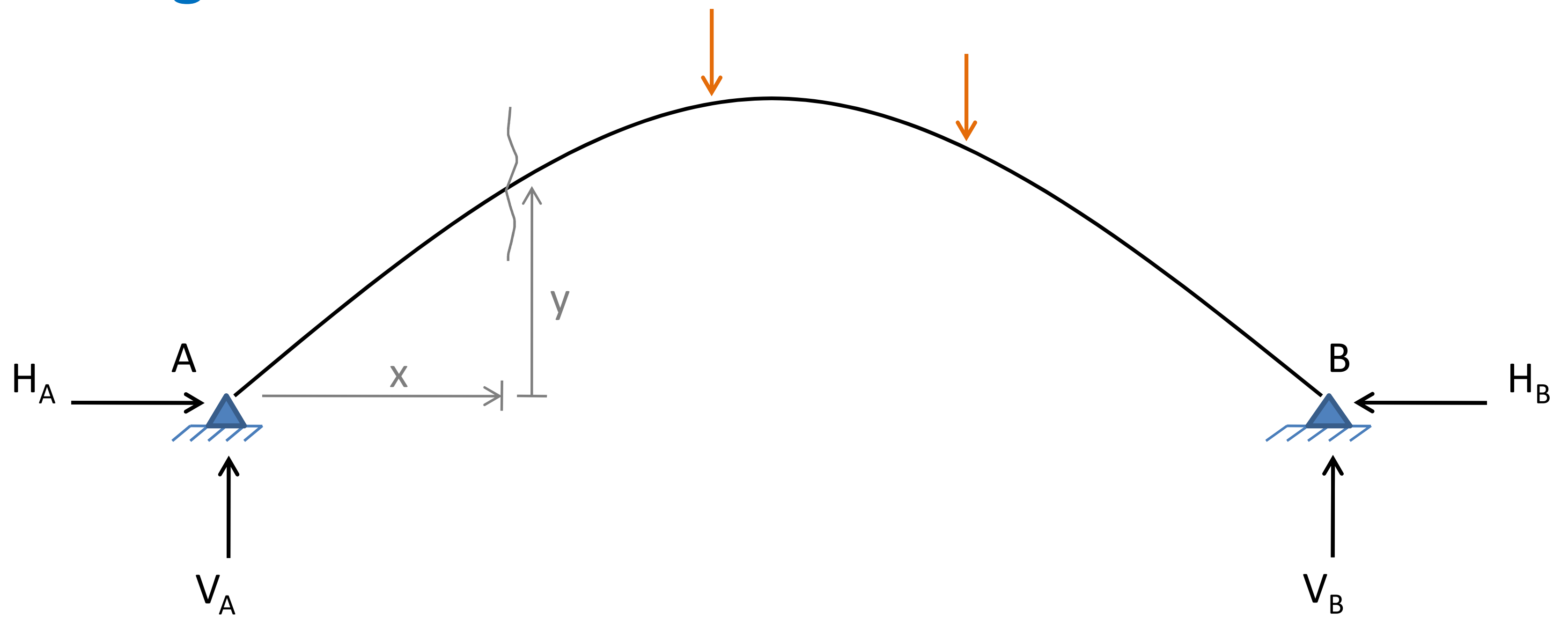
- For an Arch the shear force, S is

$$S = V_A \cos\theta - H_A \sin\theta$$

- For an Arch the Axial thrust, N is

$$N = V_A \sin\theta + H_A \cos\theta$$

Two Hinged Parabolic Arch



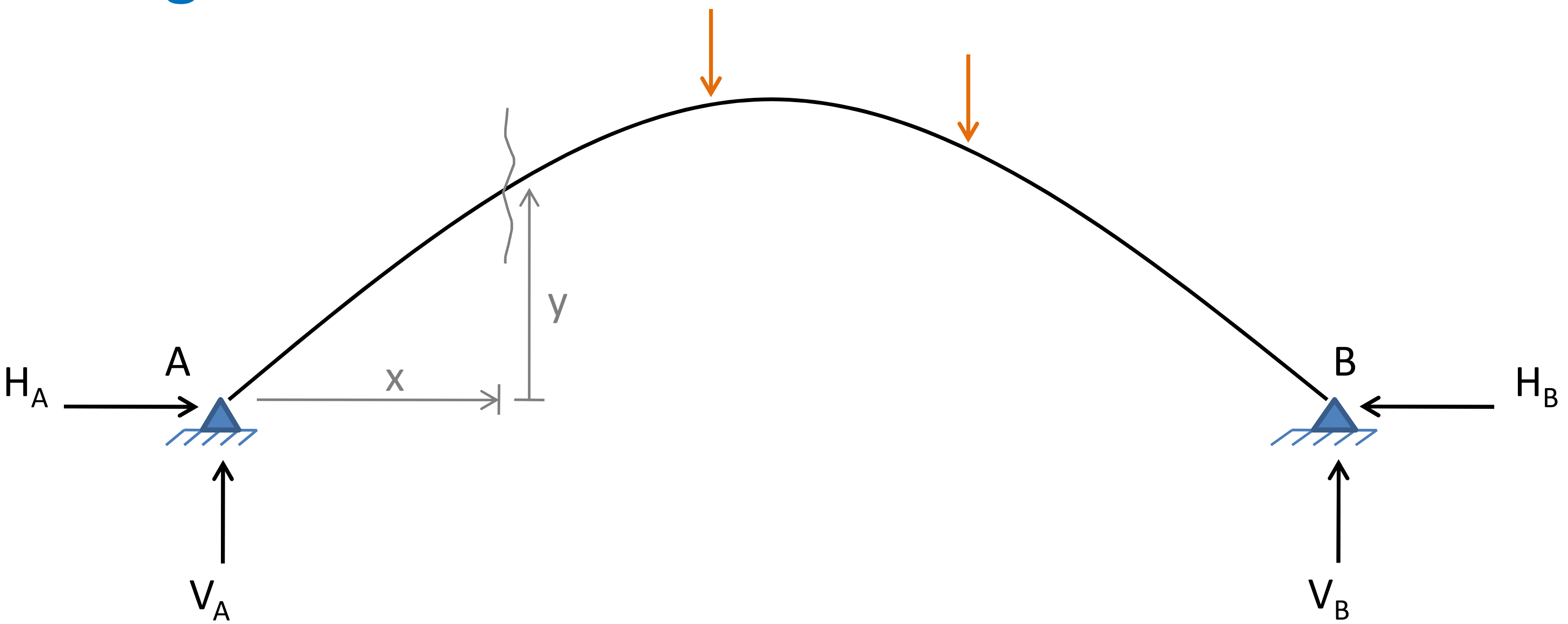
- Equation (1) becomes,

$$\frac{\partial U}{\partial H}=\int_0^L \frac{\partial M}{\partial H}\frac{\left(\mu - H_A.y\right)}{EI}ds=0$$

- If $H_A = H_B = H$ then

$$\frac{\partial U}{\partial H}=\int_0^L \frac{\partial M}{\partial H}\frac{\left(\mu - H.y\right)}{EI}ds=0$$

Two Hinged Parabolic Arch



- Partial derivative of M with respect to H is

$$\frac{\partial M}{\partial H} = -y$$

- Equation (1) becomes

$$\frac{\partial U}{\partial H} = \frac{1}{EI}\int_0^L (\mu - H.y)(-y)ds = 0$$

Two Hinged Parabolic Arch

$$\frac{\partial U}{\partial H} = -\frac{1}{EI}\int_0^L \mu y ds + \frac{H}{EI}\int_0^L y^2 ds = 0$$

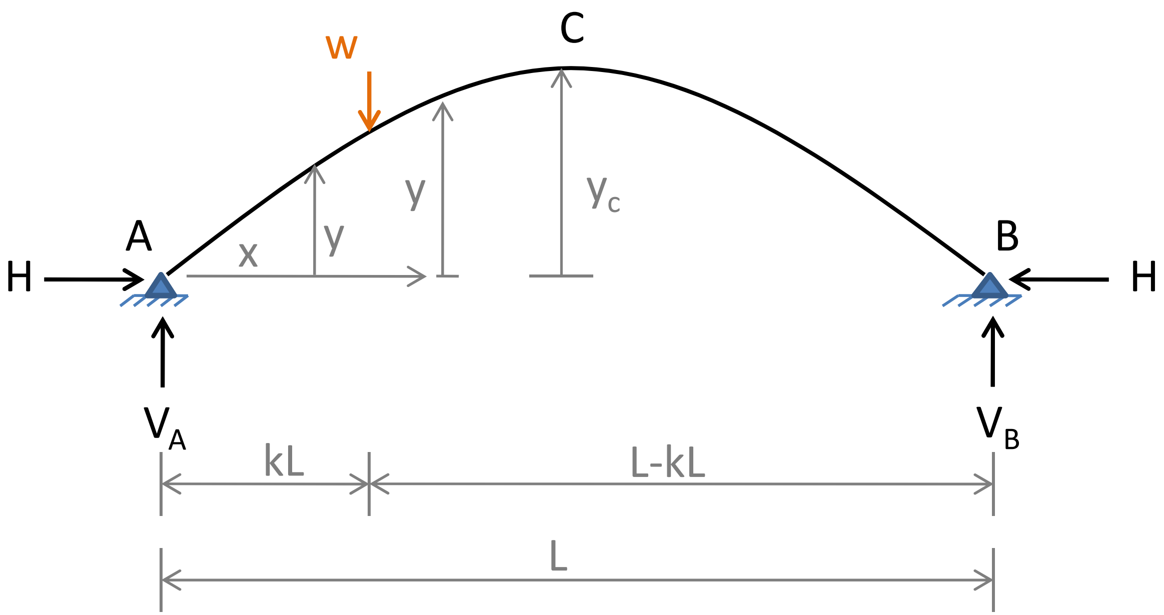
$$\frac{1}{EI}\int_0^L \mu y ds = \frac{H}{EI}\int_0^L y^2 ds$$

$$H = \frac{\int_0^L \frac{\mu y}{EI} ds}{\int_0^L \frac{y^2}{EI} ds}$$

$$H = \frac{\int_0^L \mu y ds}{\int_0^L y^2 ds}$$

Parabolic Arch Subjected to a Single Concentrated Load

$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

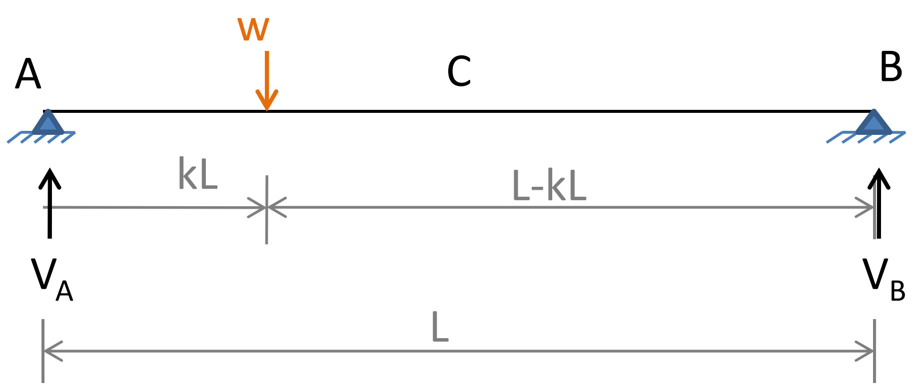


Equation of Parabola

$$y = \frac{4y_c}{L^2} x(L-x)$$

$$V_A = \frac{w(L-kL)}{L}$$

$$V_B = \frac{wkL}{L}$$

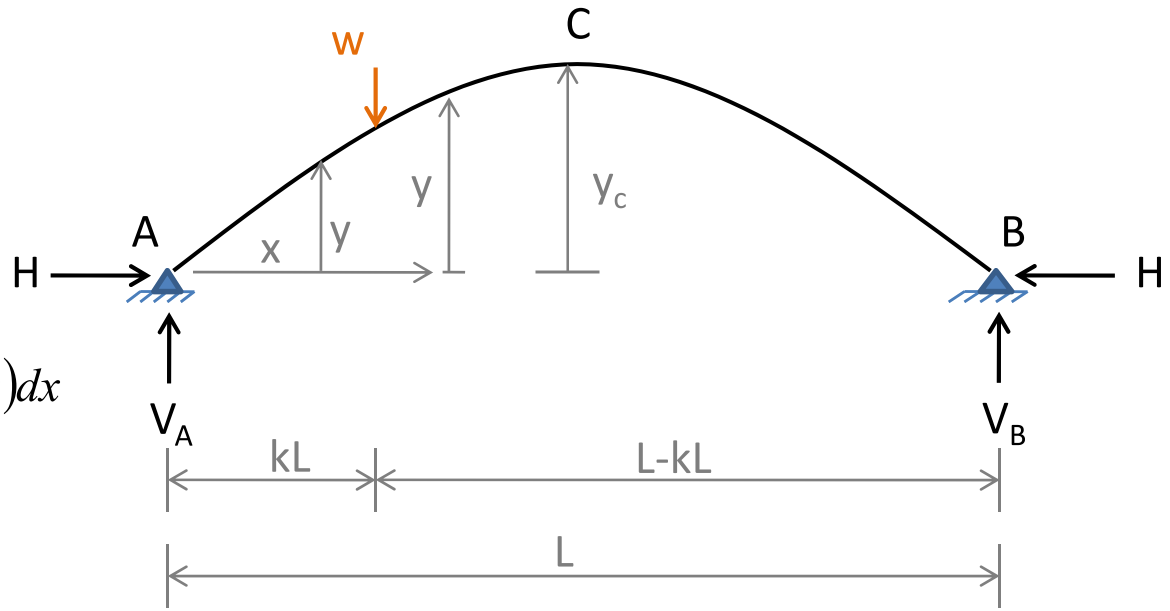


Parabolic Arch Subjected to a Single Concentrated Load

For 0 → kL

$$\mu = \frac{w(L-kL)}{L}x$$

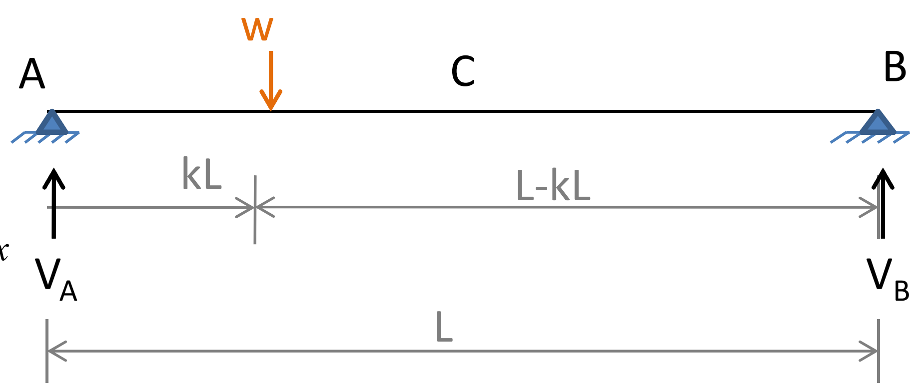
$$\int_0^{kL} \mu y dx = \int_0^{kL} \frac{w(L-kL)}{L} x \frac{4y_c x}{L^2}(L-x)dx$$



For kL → L

$$\mu = \frac{w(L-kL)}{L}x - w(x-kL)$$

$$\int_{kL}^{L} \mu y dx = \int_{kL}^{L} \left[\frac{w(L-kL)}{L}x - w(x-kL)\right]\left[\frac{4y_c}{L^2}x(L-x)\right]dx$$



Parabolic Arch Subjected to a Single Concentrated Load

For 0 → kL

$$\mu = \frac{w(L-kL)}{L}x$$

$$\int_0^{kL} \mu y dx = \int_0^{kL} \frac{w(L-kL)}{L}x\frac{4y_c x}{L^2}(L-x)dx$$

For kL → L

$$\mu = \frac{w(L-kL)}{L}x - w(x-kL)$$

$$\int_{kL}^{L} \mu y dx = \int_{kL}^{L} \left[\frac{w(L-kL)}{L}x - w(x-kL)\right]\left[\frac{4y_c}{L^2}x(L-x)\right]dx$$

Parabolic Arch Subjected to a Single Concentrated Load

$$\int_0^L y^2 dx = \int_0^L \left(\frac{4y_c}{L^2} .x(L-x) \right)^2 dx$$

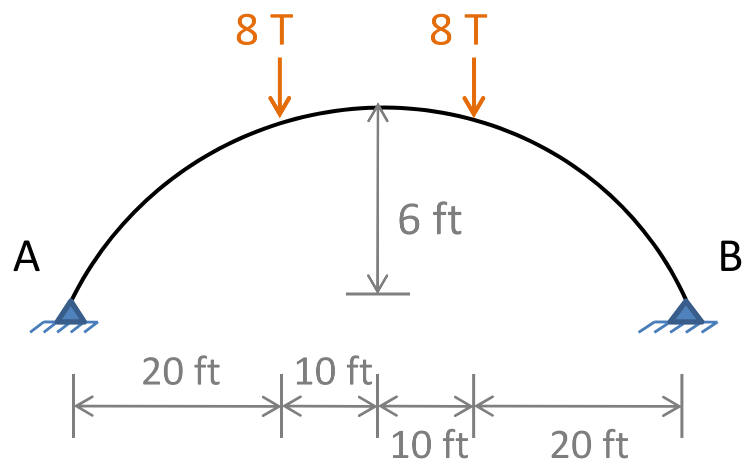
$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

$$H = \frac{\int_0^{kL} \frac{w(L-kL)}{L} x \frac{4y_c}{L^2} x(L-x) dx + \int_{kL}^{L} \left(\frac{w(L-kL)}{L} x - w(x-kL) \right) \left(\frac{4y_c}{L^2} x(L-x) \right) dx}{\int_0^L \left[\frac{4y_c}{L^2} x(L-x) \right]^2 dx}$$

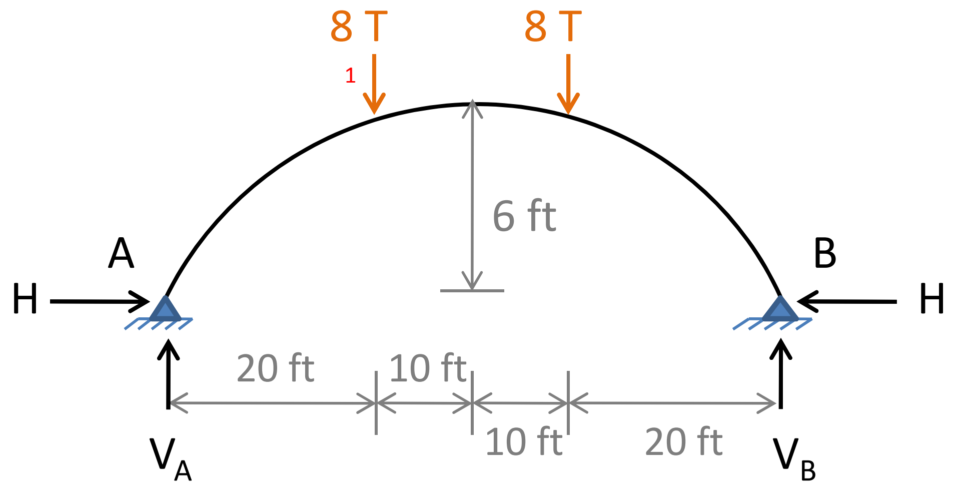
$$H = \frac{5}{8} \frac{wL}{y_c} \left(k - 2k^3 + k^4 \right)$$

Example 3

Find the horizontal thrust in the arch shown.



Solution



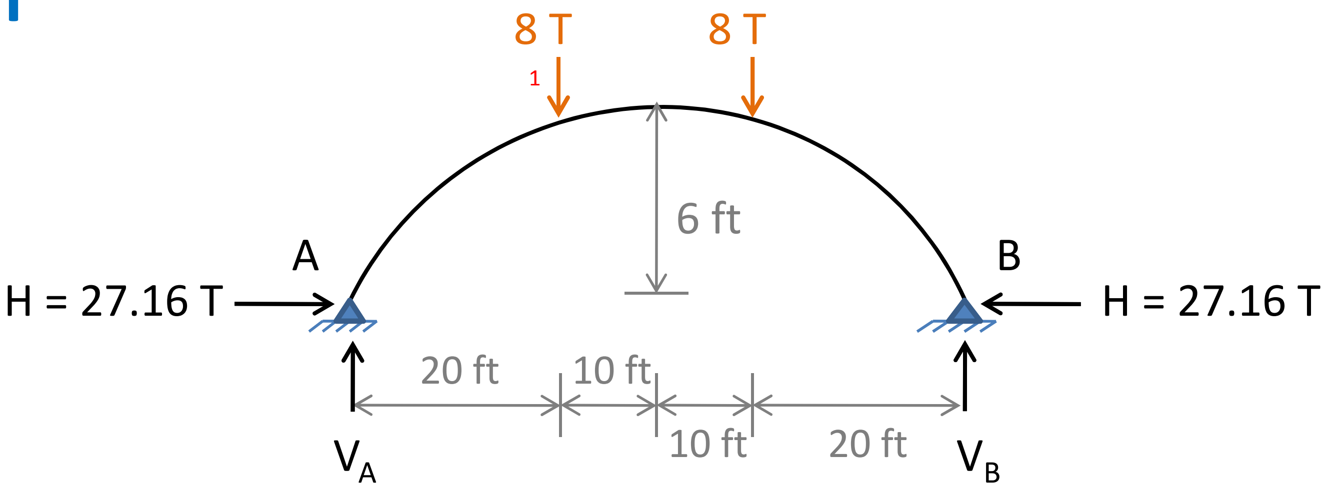
Considering Load 1

$$k = \frac{20}{60} = \frac{1}{3}$$

$$H = \frac{5}{8}\frac{wL}{y_c}\left(k - 2k^3 + k^4\right)$$

$$= \frac{5}{8} \times \frac{8 \times 60}{6} \times \left(\frac{1}{3} - 2\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4\right)$$

Solution



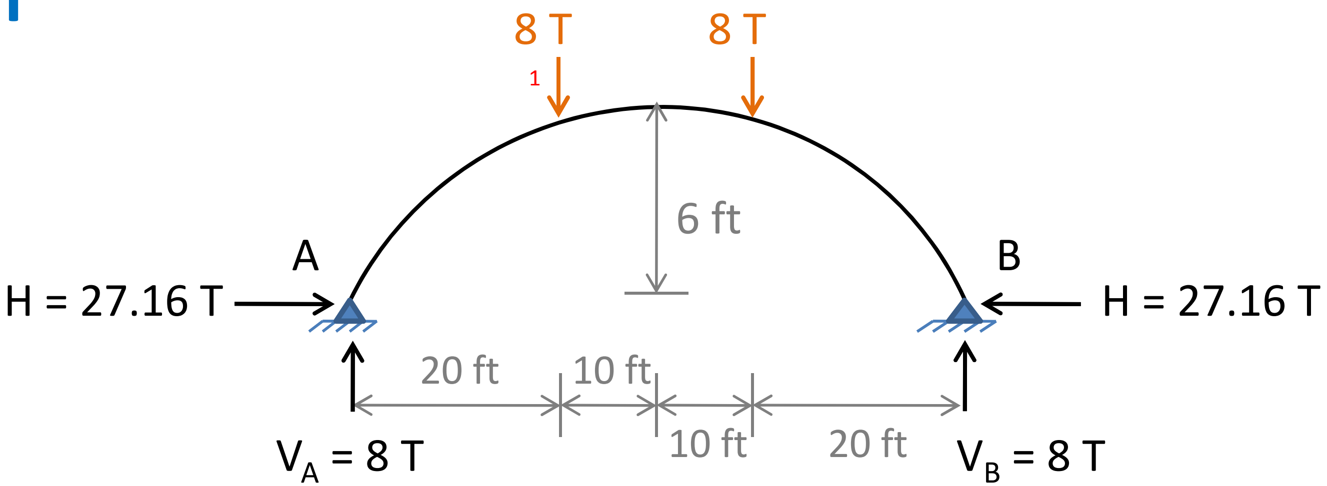
$$= 50\left(\frac{1}{3} - \frac{2}{27} + \frac{1}{81}\right)$$

$$H = 13.58\ \mathrm{T}$$

As the load is symmetric, so total reaction will be

$$H = 2 \times 13.58 = 27.16\ \mathrm{T}$$

Solution



To find V_A and V_B, apply equations of equilibrium

$$\sum M_B = 0$$

$$V_A \times 60 - 8 \times 40 - 8 \times 20 = 0$$

$$V_A = 8\,T$$

$$\sum F_y = 0$$

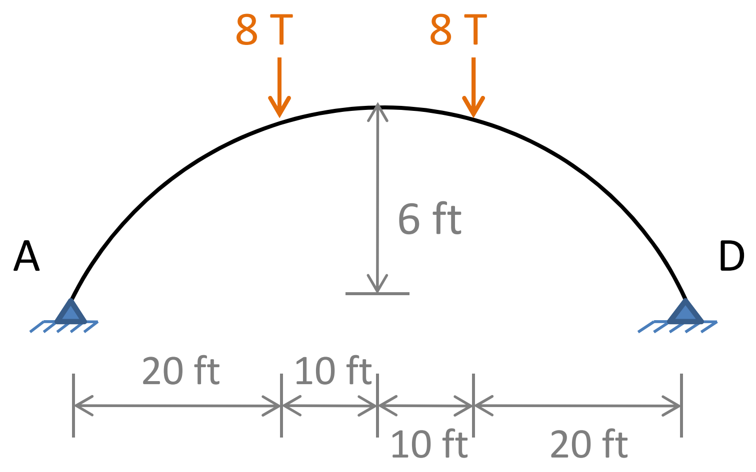
$$V_A + V_B - 8 - 8 = 0$$

$$8 + V_B - 8 - 8 = 0$$

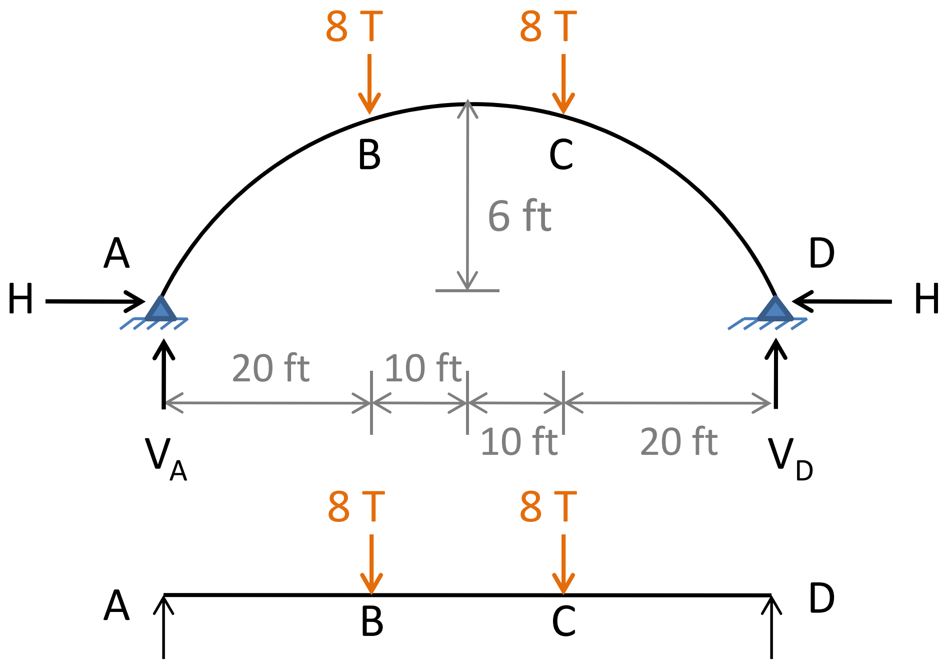
$$V_B = 8\,T$$

Example 3

Find the horizontal thrust in the arch shown.



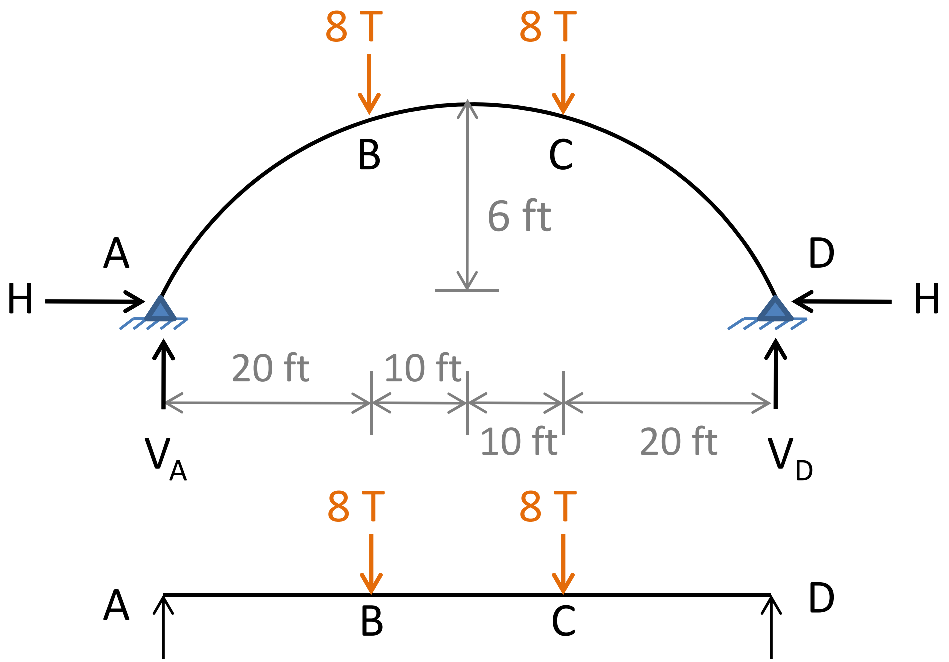
Solution



$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

$$y = \frac{4y_c}{L^2} x(L-x) = \frac{4 \times 6}{60^2} .x.(60-x) = \frac{x}{150}(60-x)$$

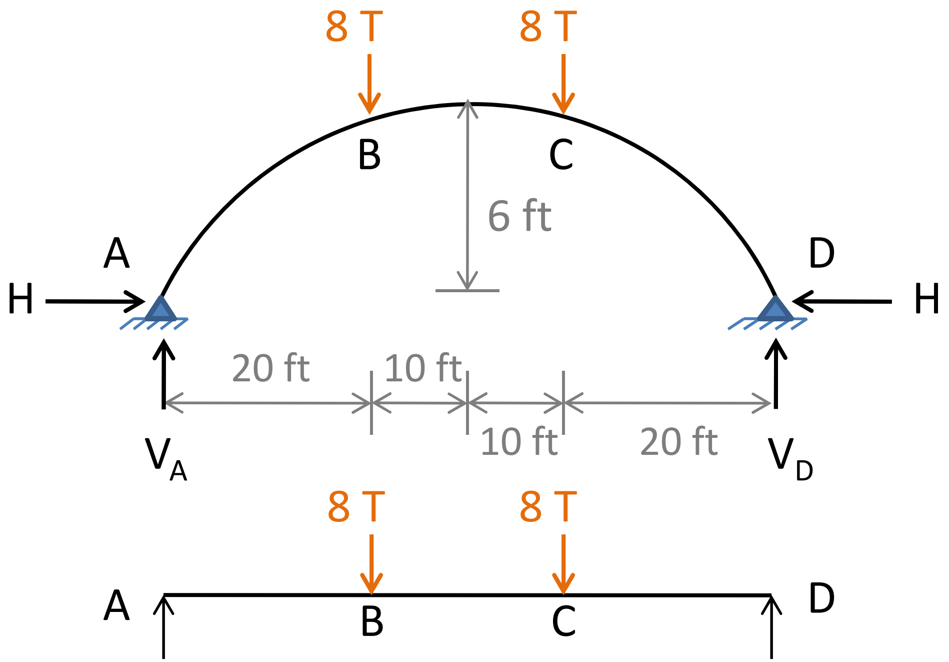
Solution



Portion	Origin	Limits	μ
AB	A	0 – 20	8x
BC	A	20 – 40	8x – 8(x-20)
CD	D	0 – 20	8x

Solution

$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

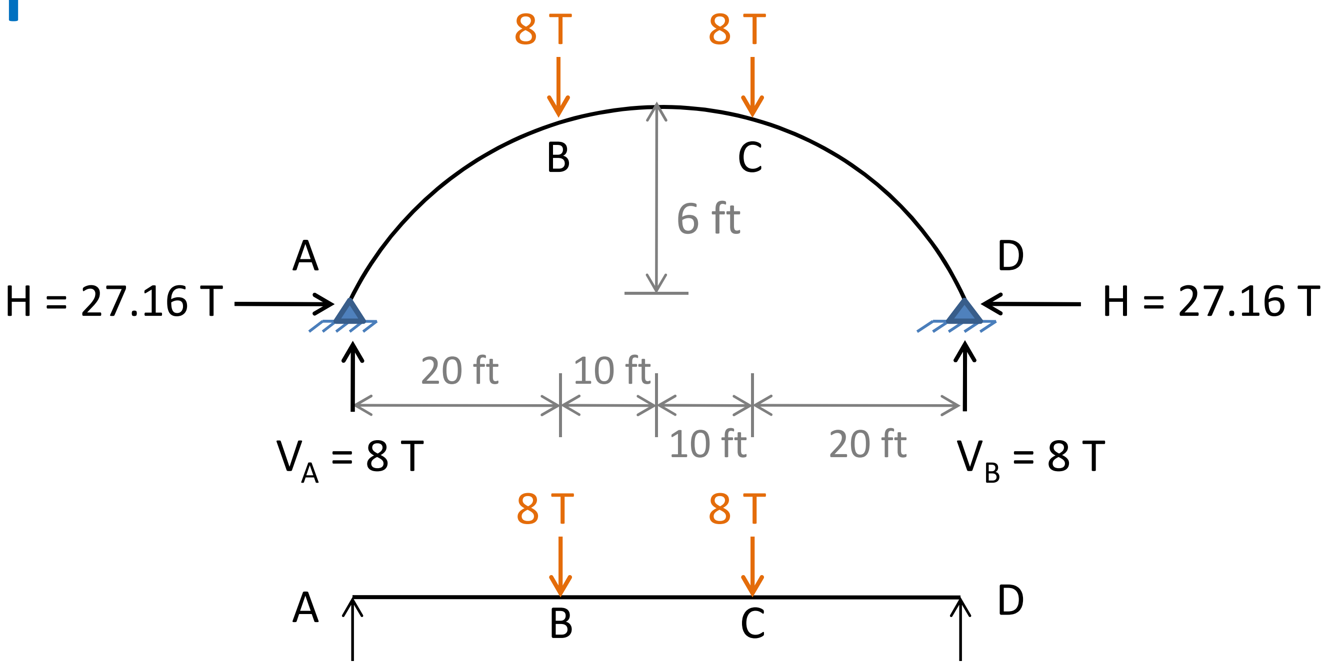


$$\int_0^L \mu . y dx = 2\int_0^{20} 8x . \frac{x}{150}(60-x)dx + \int_0^{20}(8x-8(x-20)) . \frac{x}{150}(60-x)dx$$

$$= 31288.7 \ T\text{-}ft^3$$

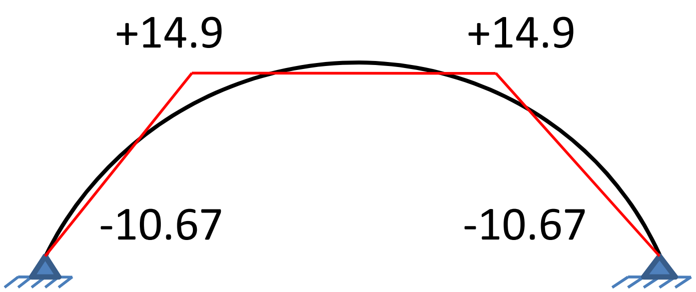
$$\int_0^L y^2 dx = \int_0^{60}\left(\frac{x}{150}(60-x)\right)^2 dx = 1152 \ ft^3$$

Solution



$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx} = \frac{31288.7}{1152} = 27.16\,T$$

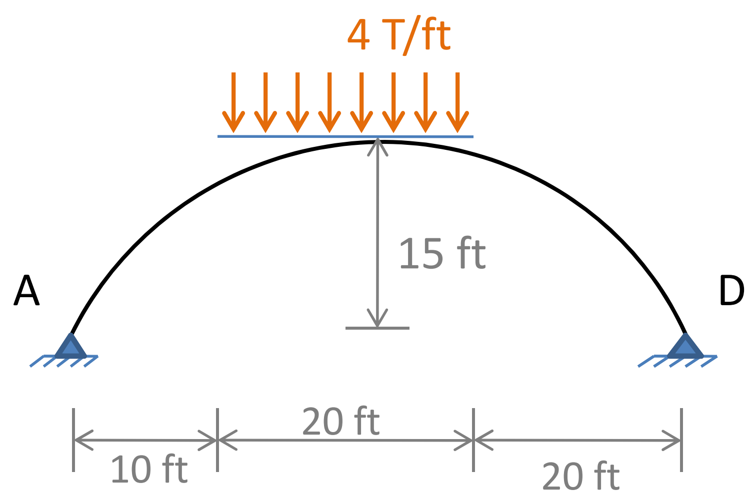
Solution



Bending Moment Diagram

Example 4

Analyze the arch shown.



Solution

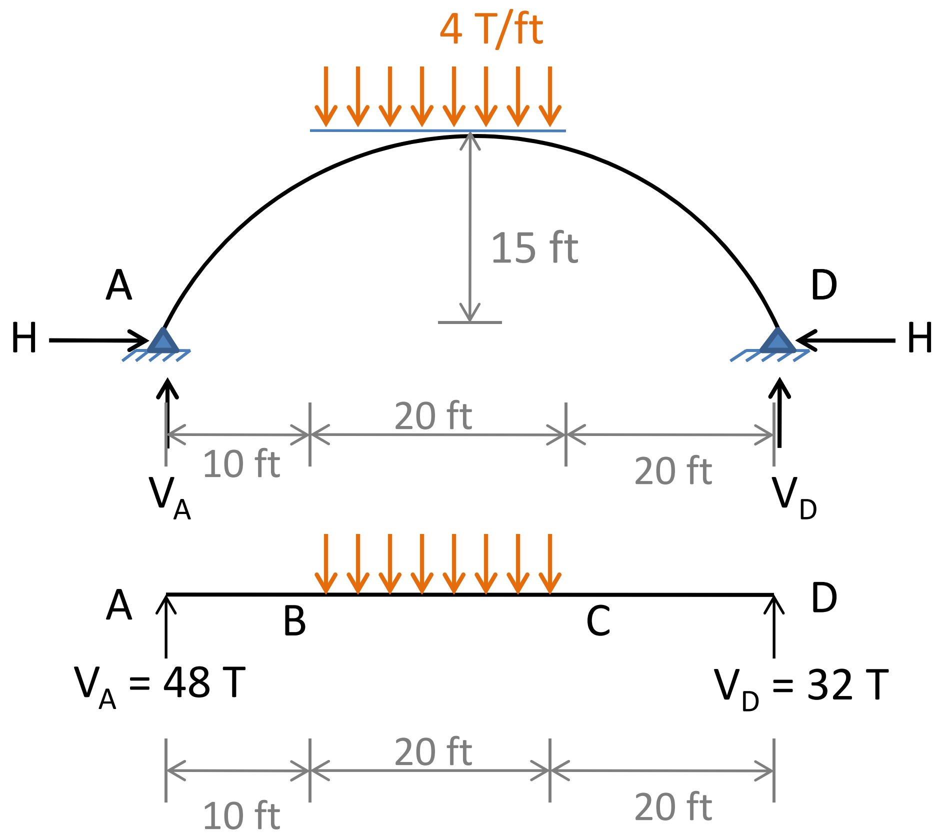
$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

$$y = \frac{4y_c}{L^2} x(L-x)$$

$$y = \frac{4 \times 15}{50^2} x(50-x)$$

$$y = \frac{4 \times 3}{500} x(50-x)$$

$$y = \frac{12}{500} x(50-x)$$

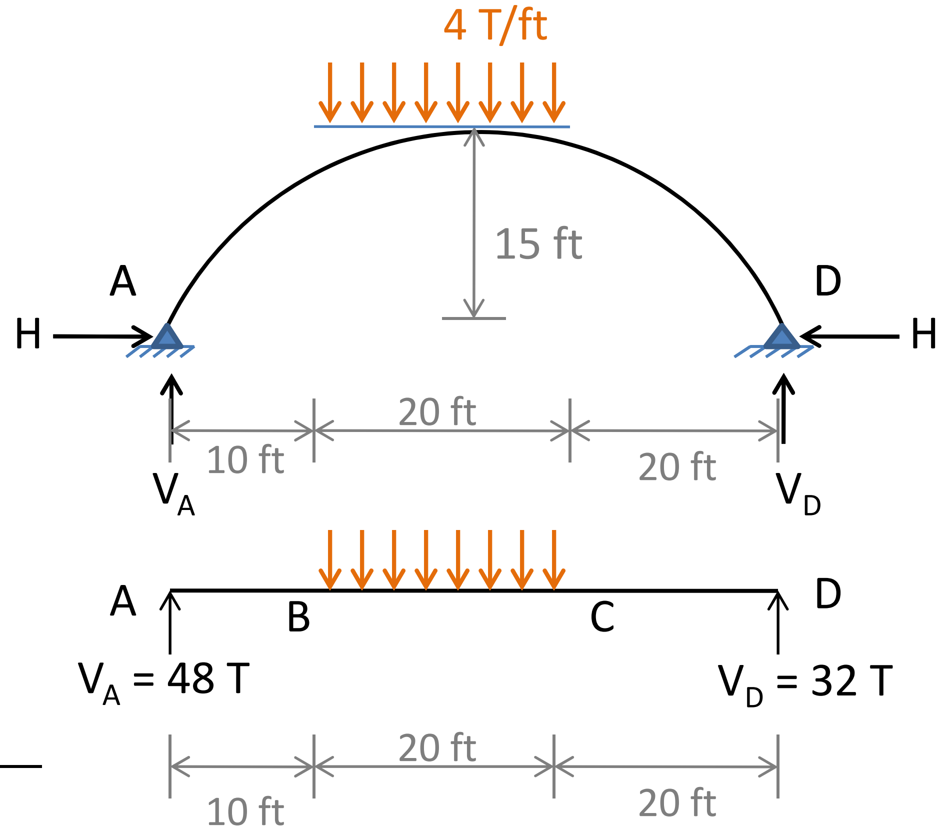


μ= bending moment while considering
simply supported beam
y = equation of parabola

Solution

$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

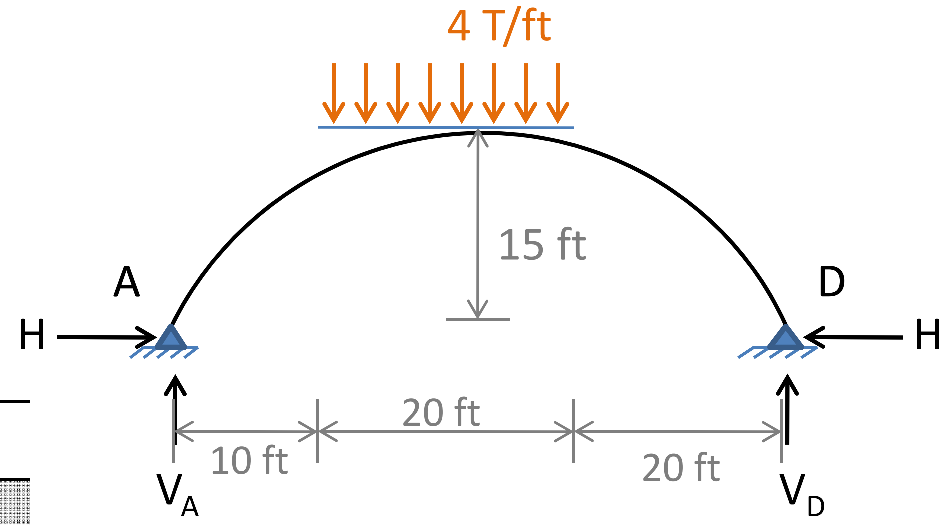
μ= bending moment while considering simply supported beam
y = equation of parabola



Portion	Origin	Limits	μ
AB	A	0 – 10	48x
BC	A	10 – 30	$48x - 4(x-10)^2/2$
DC	D	0 – 20	32x

Solution

$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx} \qquad y = \frac{12}{500}x(50-x)$$



Portion	Origin	Limits	μ
AB	A	0 – 10	48x
BC	A	10 – 30	48x – 4(x-10)²/2
DC	D	0 – 20	32x

$$\int_0^L \mu . y dx = \int_0^{10} (48x)\left(\frac{12}{500}x(50-x)\right)dx + \int_{10}^{30}\left(48x - 2(x-10)^2\right)\frac{12}{500}x(50-x)dx$$

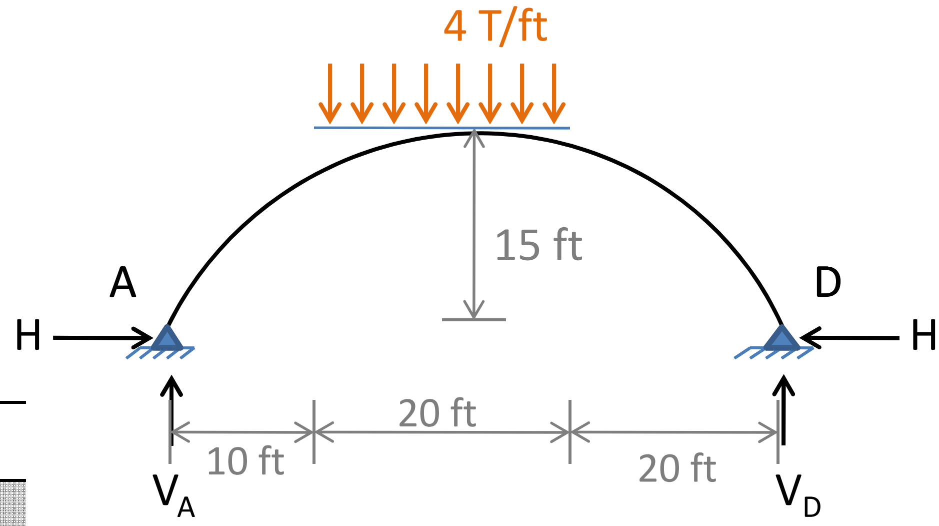
$$+\int_0^{20}(32x)\left(\frac{12}{500}x(50-x)\right)dx$$

$$= 16320 + 190720 + 71680$$

$$= 278720 \ T\text{-}ft^3$$

Solution

$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx} \qquad y = \frac{12}{500}x(50-x)$$



Portion	Origin	Limits	μ
AB	A	0 – 10	48x
BC	A	10 – 30	48x – 4(x-10)²/2
DC	D	0 – 20	32x

$$\int_0^L y^2 dx = \int_0^{50}\left(\frac{12}{500}x(50-x)\right)^2 dx$$

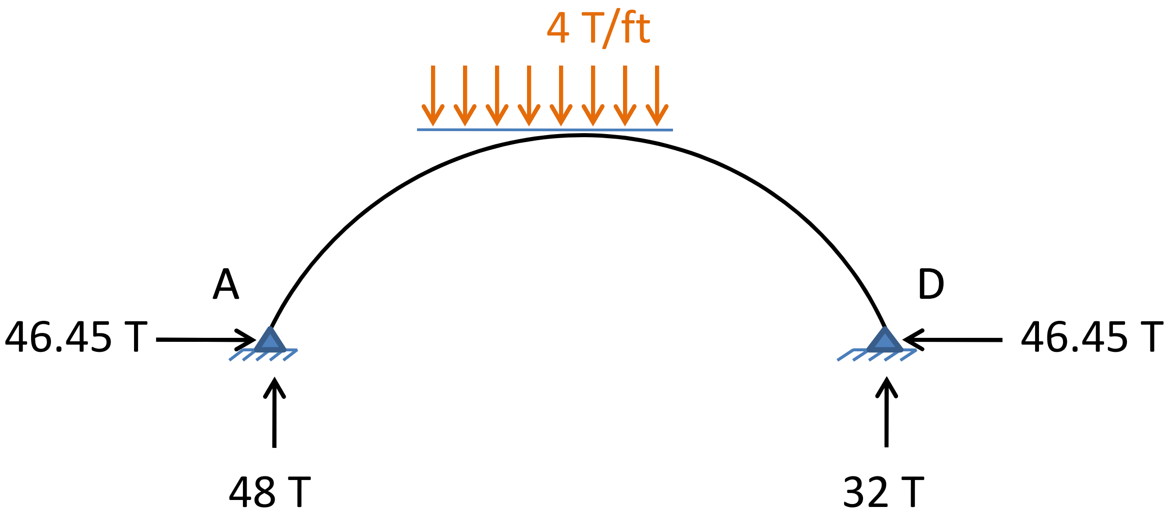
$$= 6000\ ft^3$$

Solution

put these values into the following equation

$$H = \frac{\int_0^L \mu y dx}{\int_0^L y^2 dx}$$

$$H = \frac{278720}{6000} = 46.45\,T$$



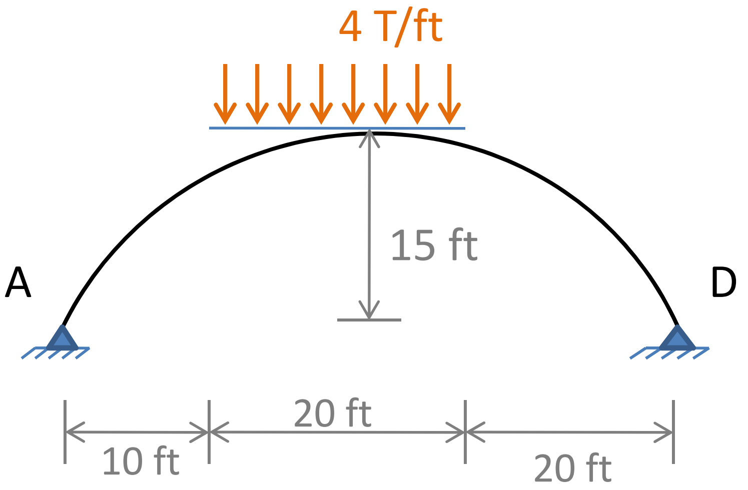
Method of Summation

- The arch rib is divided into a number of equal parts of length δ_s.
- The values of M and y are found at mid points of each of these lengths and the evaluation of numerator and denominator of expression for H is carried out by addition

$$H = \frac{\sum \mu y \delta_s}{\sum y^2 \delta_s}$$

Example 4

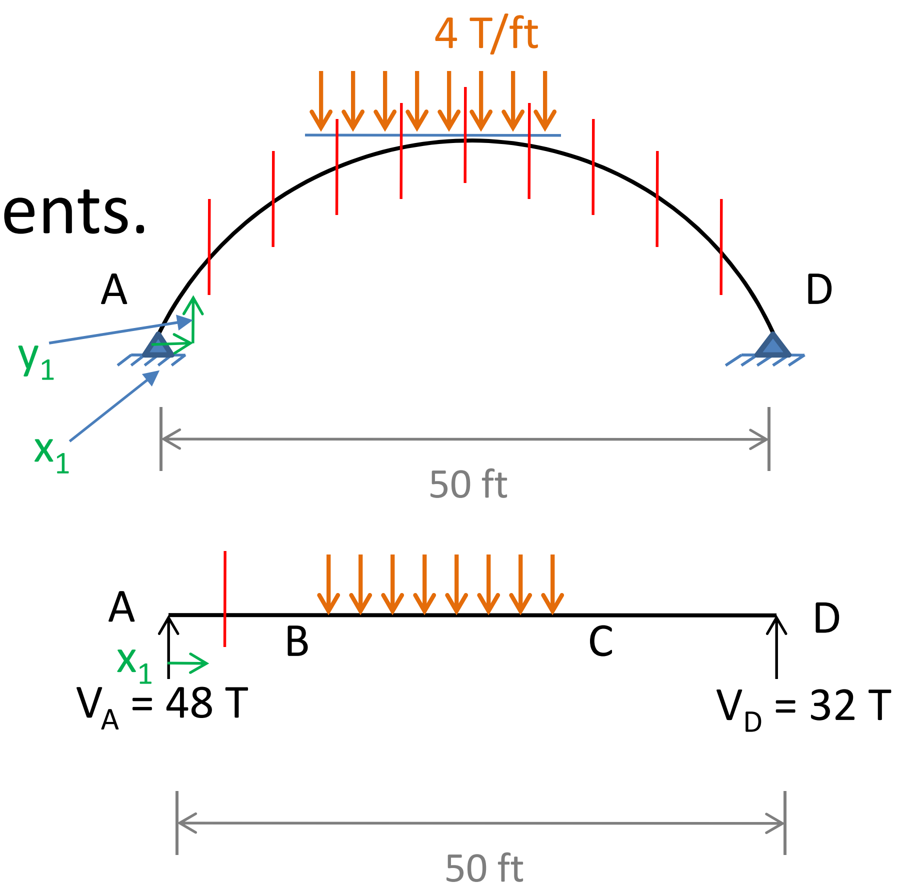
Analyze the arch shown by the method of summation.



Solution

$$H = \frac{\sum \mu y \delta_x}{\sum y^2 \delta_x}$$

Divide the arch into equal segments.



$$\delta_s = \frac{L}{10} = \frac{50}{10} = 5\ ft$$

$$x_1 = 2.5\ ft$$

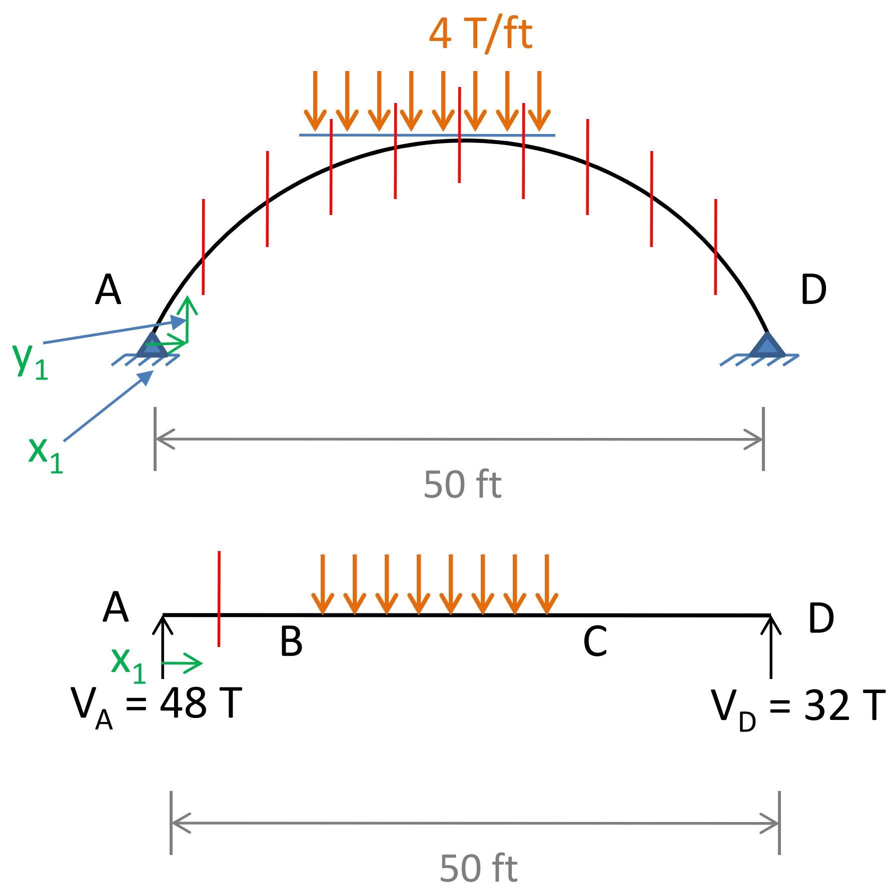
$$y_1 = \frac{4y_c}{L^2} x(L-x) = \frac{4 \times 15}{2500} x(L-x)$$

$$= \frac{60}{2500} \times 2.5 \times (50 - 2.5) = 2.85\ ft$$

$$\mu_1 = V_A \times x_1 = 48 \times 2.5 = 120\ T - ft$$

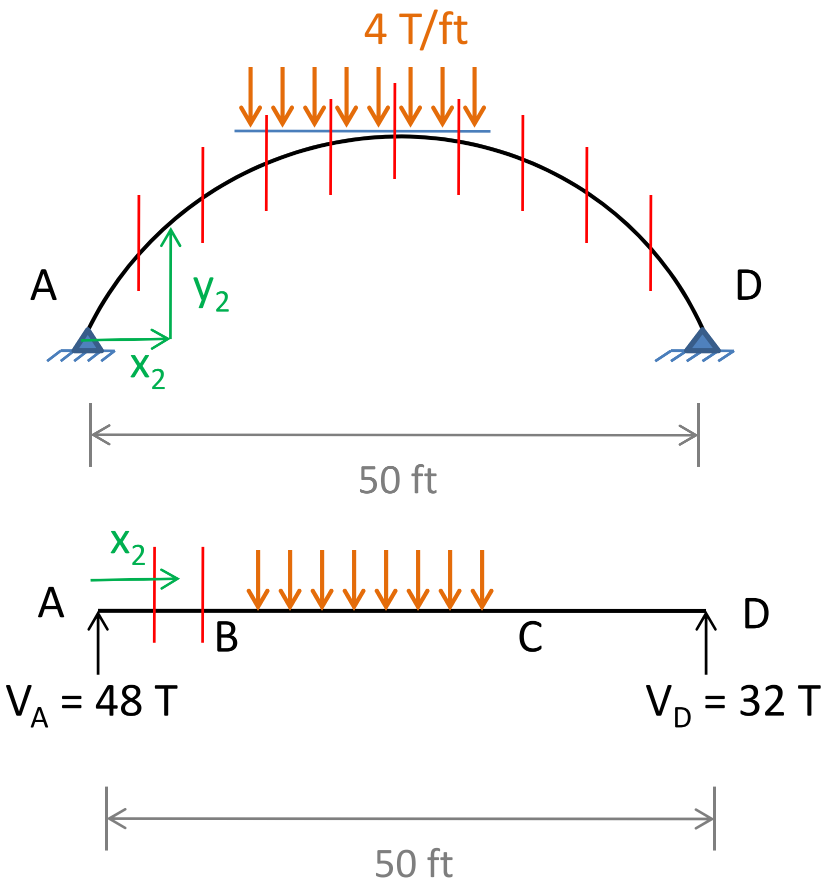
Solution

Portion	x (ft)	y (ft)	µ (T-ft)	y^2	µy
1	2.5	2.85	120	8.1	344



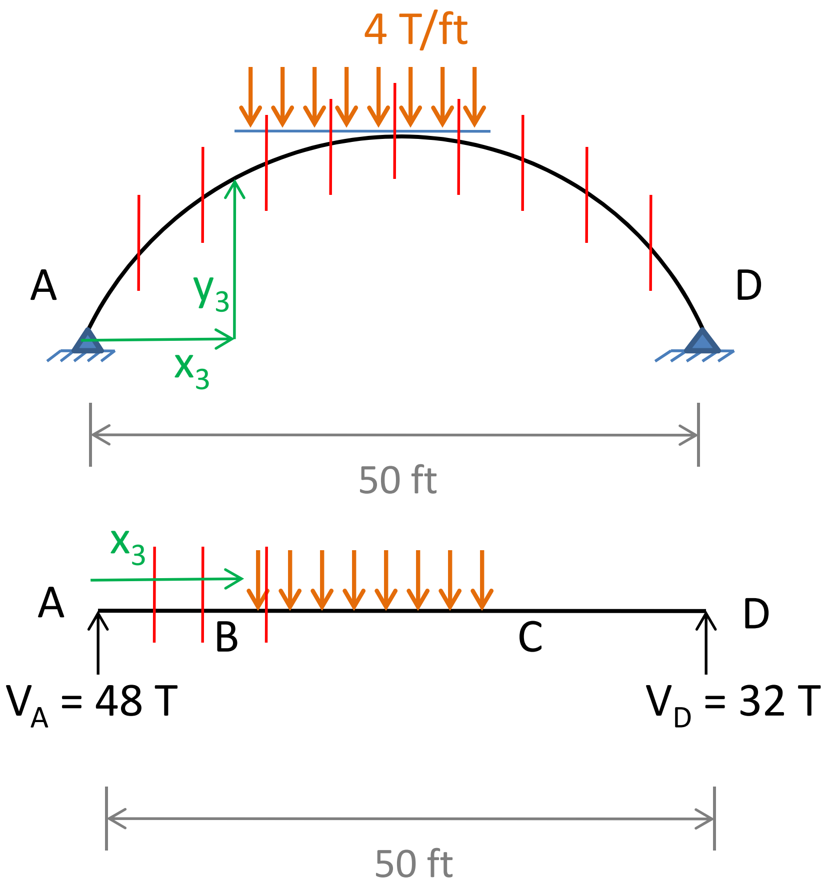
Solution

Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750



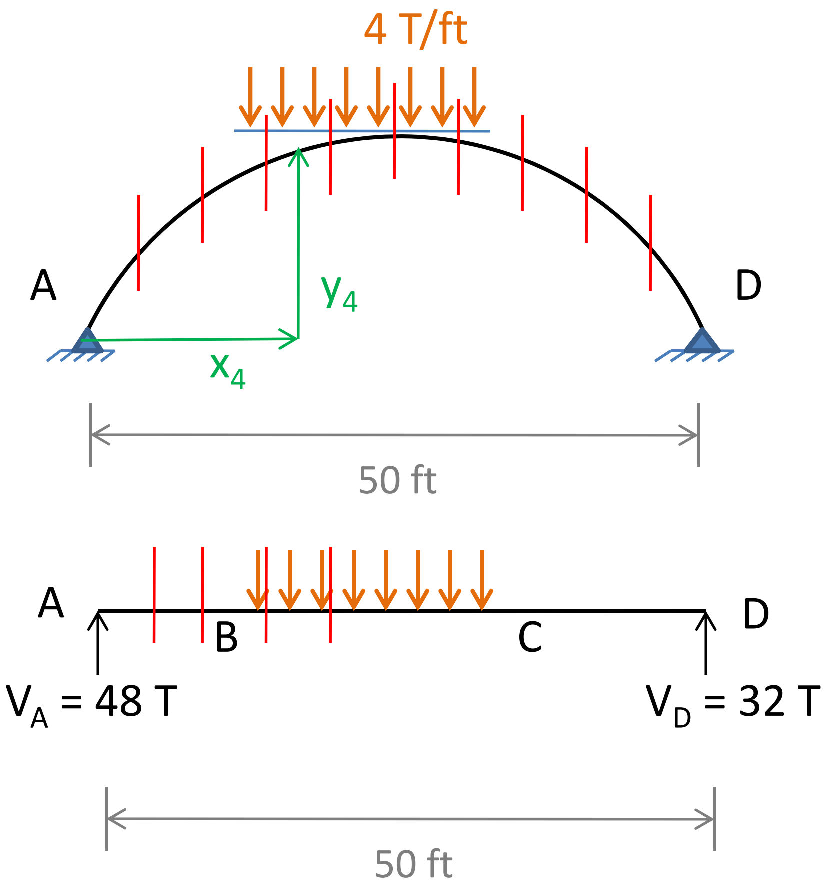
Solution

Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600



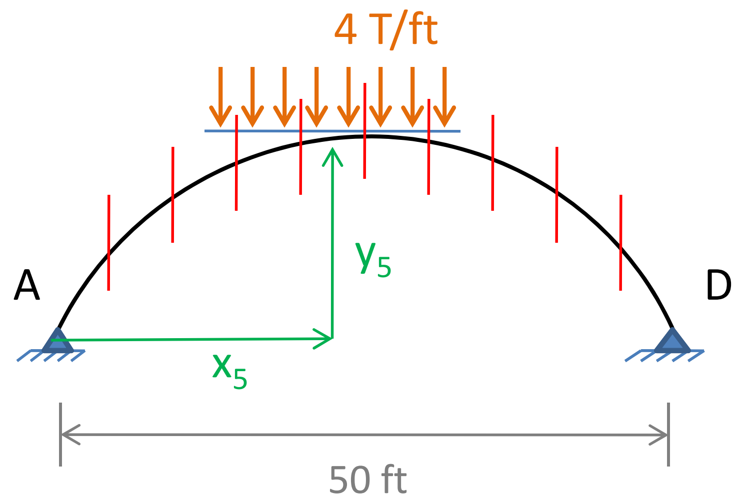
Solution

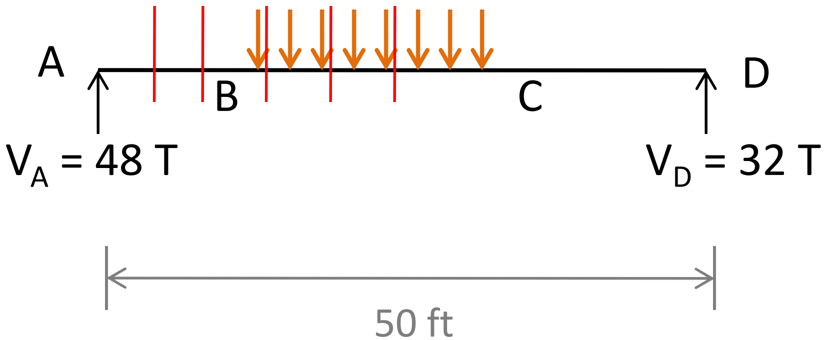
Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930



Solution

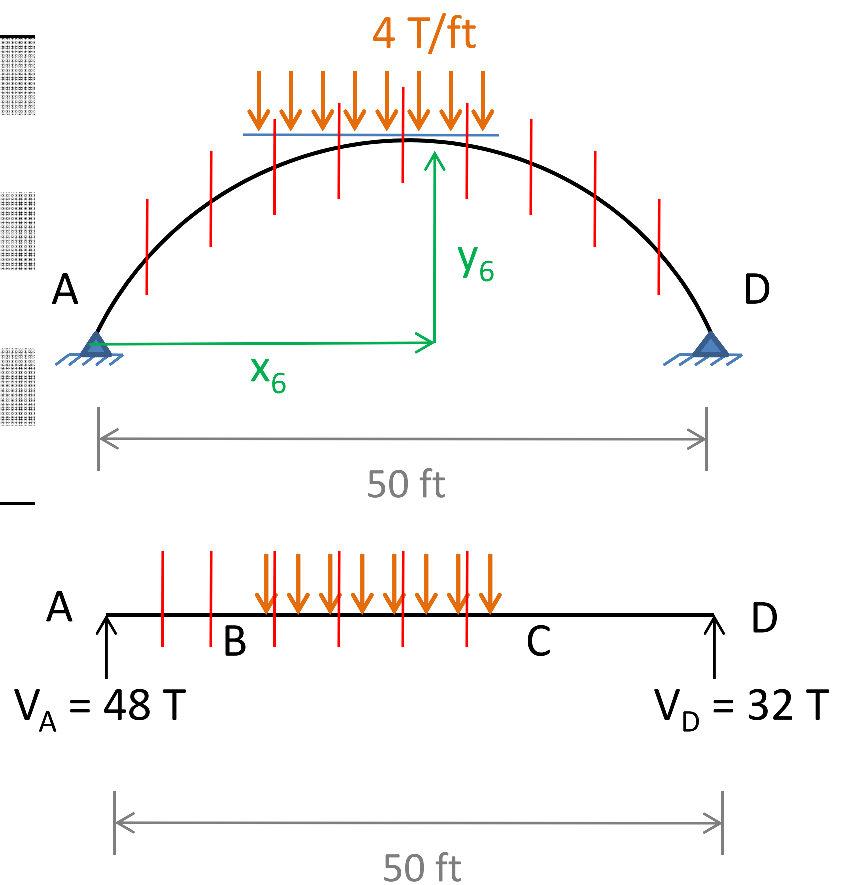
Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930
5	22.5	14.85	767	220.6	11400





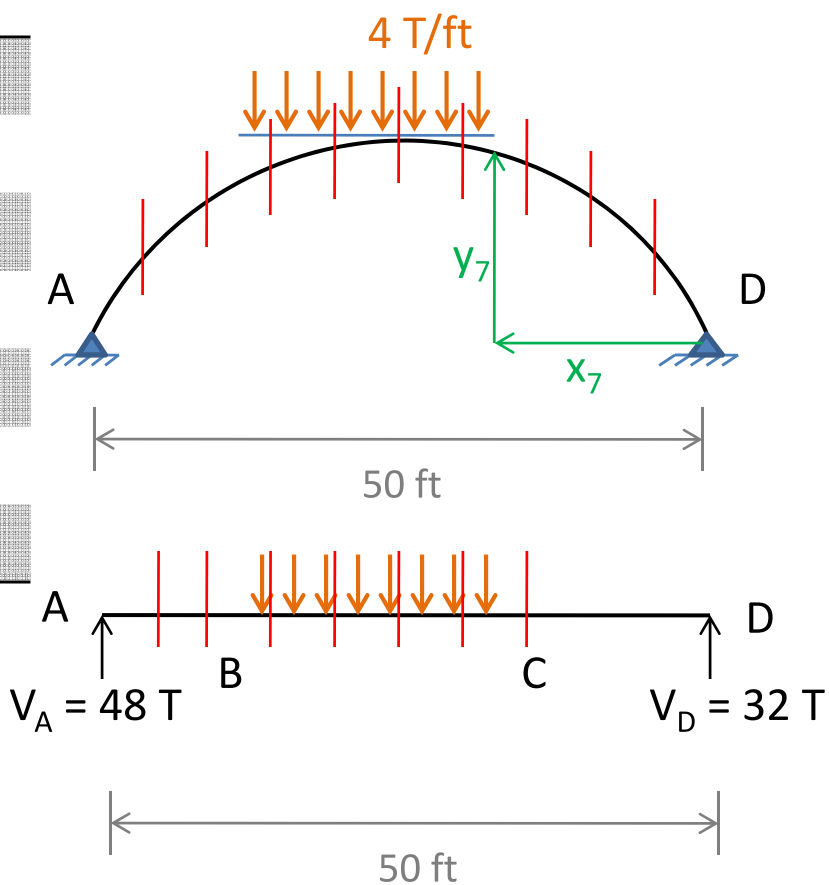
Solution

Portion	x (ft)	y (ft)	µ (T-ft)	y^2	µy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930
5	22.5	14.85	767	220.6	11400
6	27.5	14.85	707	220.6	10500



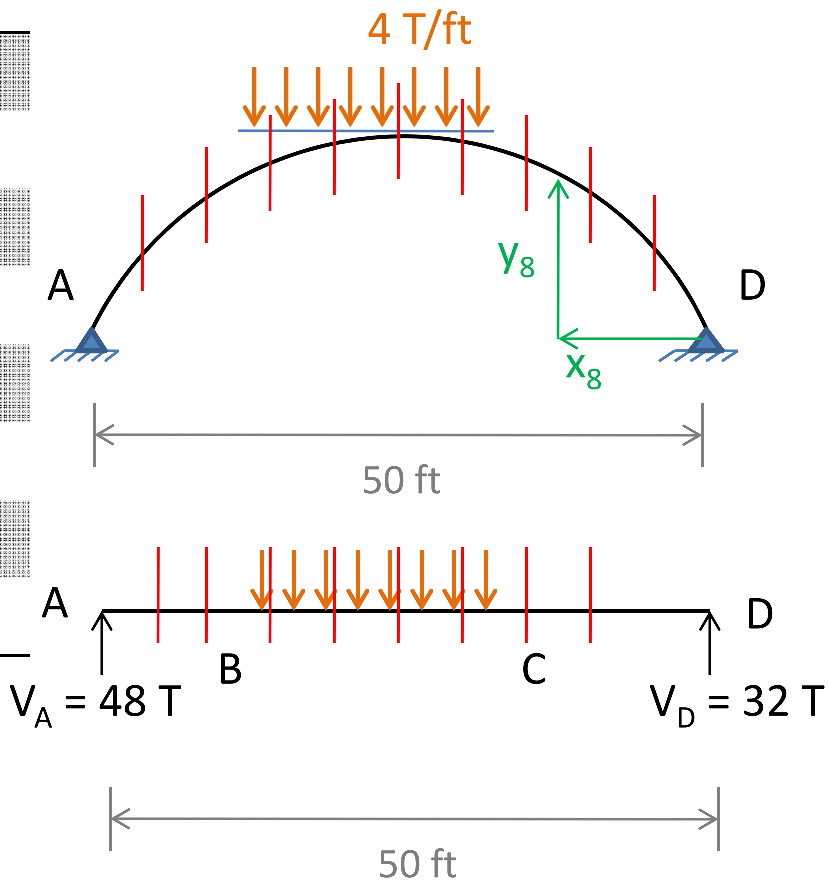
Solution

Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930
5	22.5	14.85	767	220.6	11400
6	27.5	14.85	707	220.6	10500
7	17.5	13.65	560	186.3	7650



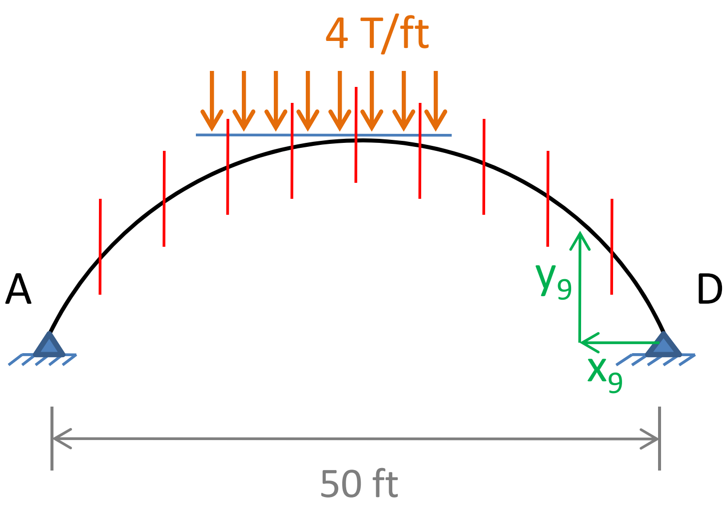
Solution

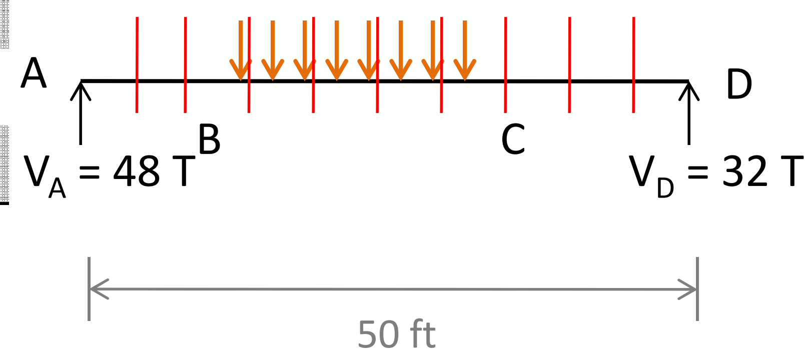
Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930
5	22.5	14.85	767	220.6	11400
6	27.5	14.85	707	220.6	10500
7	17.5	13.65	560	186.3	7650
8	12.5	11.25	400	126.7	4500



Solution

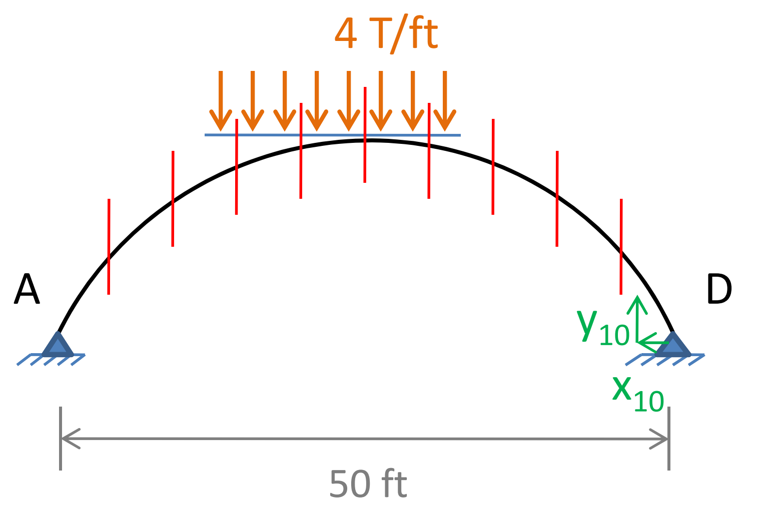
Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930
5	22.5	14.85	767	220.6	11400
6	27.5	14.85	707	220.6	10500
7	17.5	13.65	560	186.3	7650
8	12.5	11.25	400	126.7	4500
9	7.5	7.65	240	58.5	1835

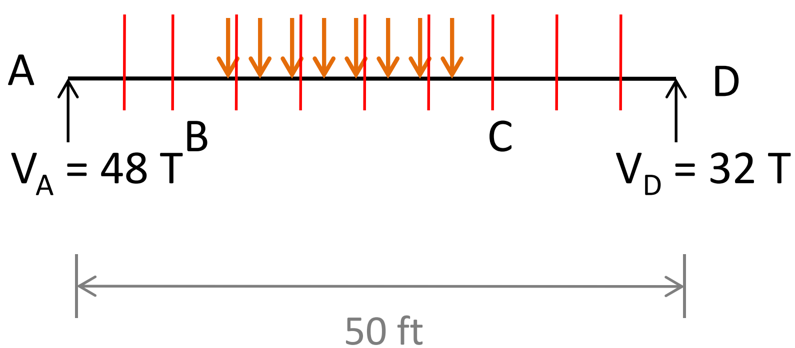




Solution

Portion	x (ft)	y (ft)	μ (T-ft)	y^2	μy
1	2.5	2.85	120	8.1	344
2	7.5	7.65	360	58.5	2750
3	12.5	11.25	587	126.7	6600
4	17.5	13.65	727	186.3	9930
5	22.5	14.85	767	220.6	11400
6	27.5	14.85	707	220.6	10500
7	17.5	13.65	560	186.3	7650
8	12.5	11.25	400	126.7	4500
9	7.5	7.65	240	58.5	1835
10	2.5	2.85	80	8.1	228
			∑	1200.4	55737



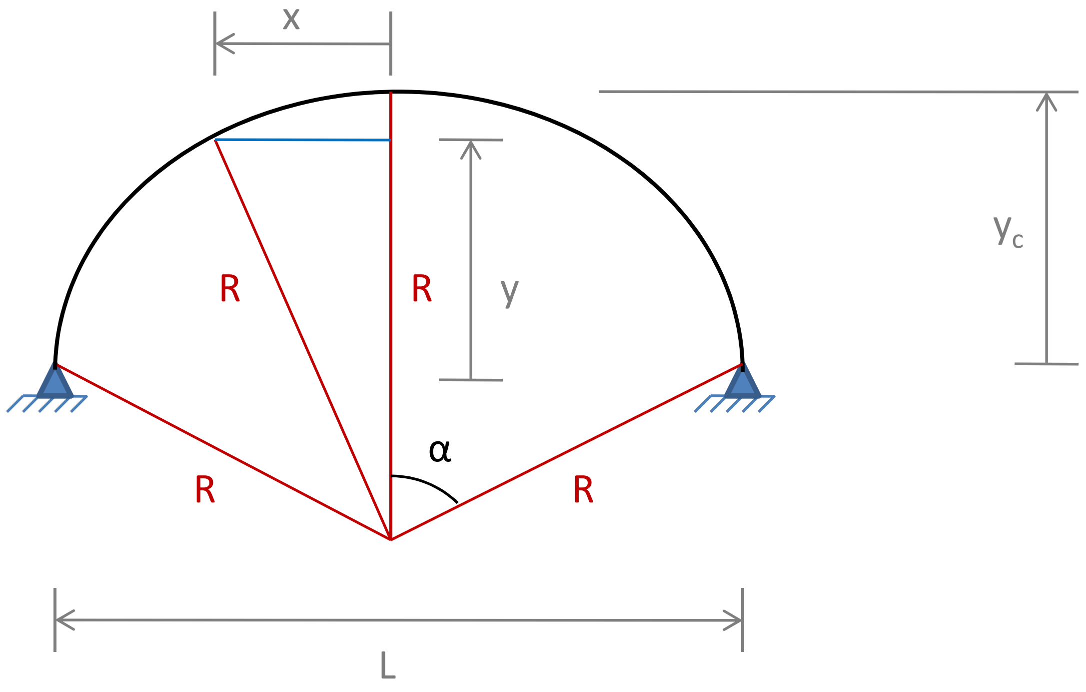


$$H = \frac{\sum \mu y \delta_x}{\sum y^2 \delta_x} = \frac{55,737}{1200.4} = 46.3\,T$$

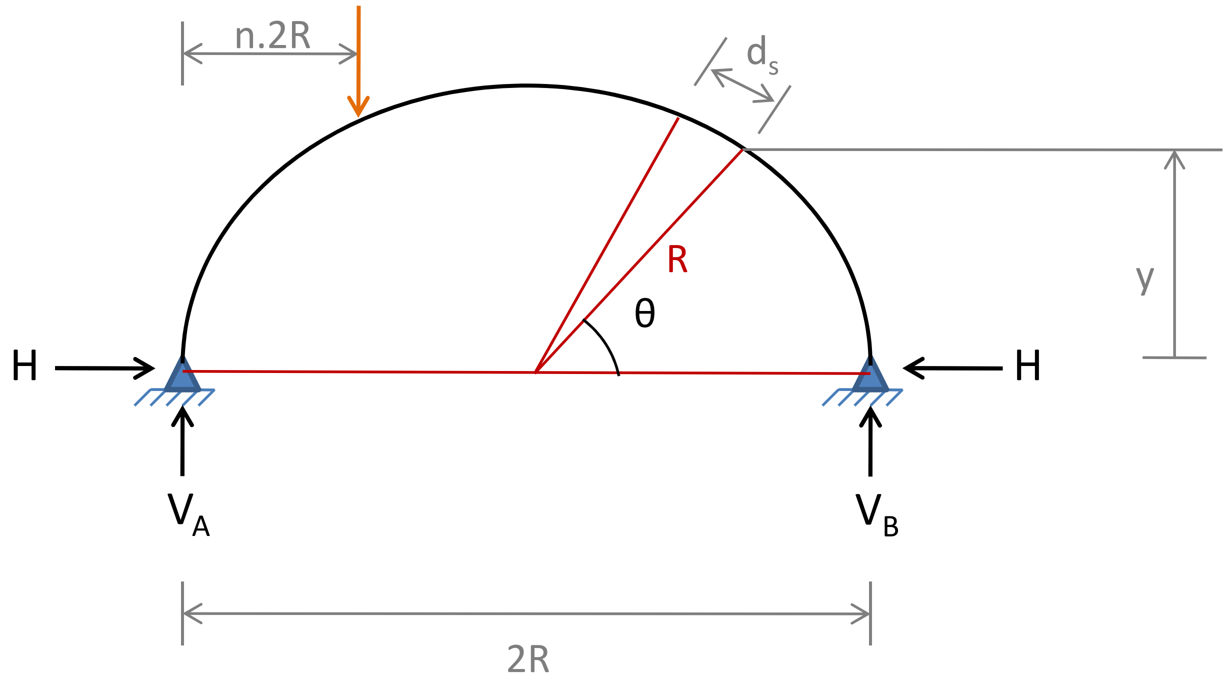
Circular Arch

- The circular arches are a portion of circle.
- Usually span & central rise are given.
- Consider the arch shown in Figure below

$$y = \sqrt{R^2 - x^2} - (R - y_c)$$



Semi-Circular Arch



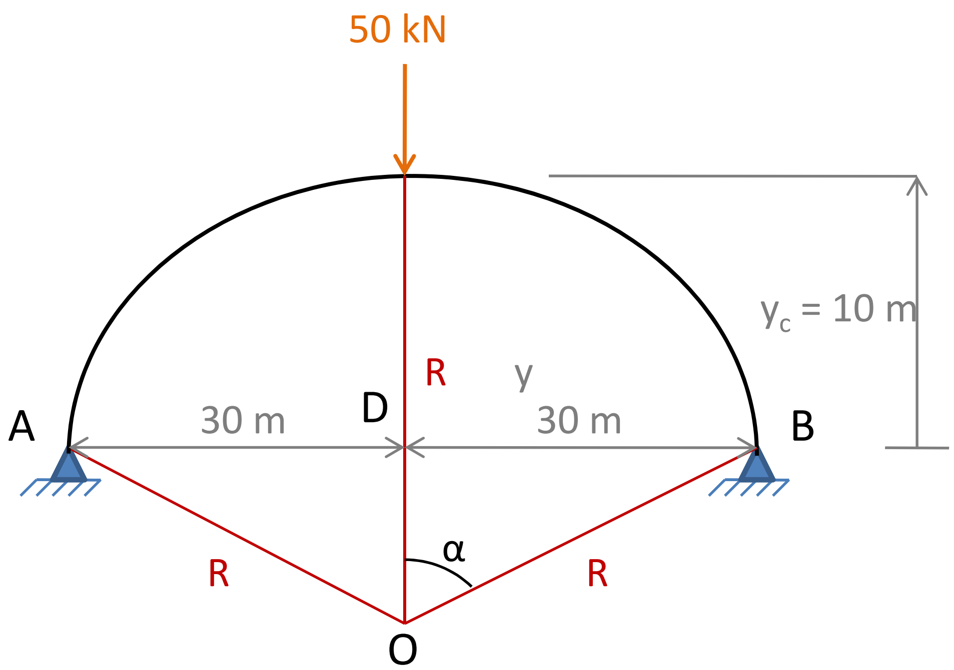
$$U = \int_0^L \frac{M^2}{2EI} dx \qquad y = R\sin\theta \qquad H = \frac{4wn}{\pi}(1-n)$$

$$H = \frac{\int \mu y ds}{\int y^2 ds} \qquad ds = Rd\theta$$

For UDL $H = \dfrac{4wR}{3\pi}$

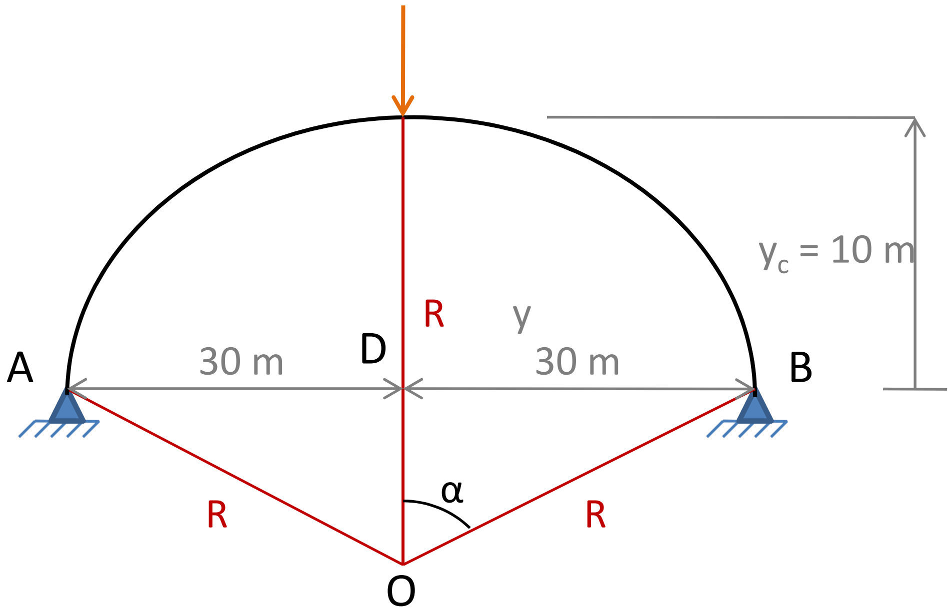
Example 5

Two hinged circular arch carries a concentrated load of 50 kN at the crown. The span and rise are 60m and 10m respectively. Find the horizontal thrust at the abutment.



Solution

Consider triangle OBD



$$R^2 = (R - y_c)^2 + (30)^2$$

$$R = 50\ m$$

$$\sin\alpha = \frac{30}{50}$$

$$\alpha = 36.87^\circ = 0.6435\ rad$$

$$S = Length\ of\ arch = R2\alpha = 64.35\ m$$

$$H = \frac{\int \mu y ds}{\int y^2 ds}$$

Solution

$$H = \frac{\int \mu y ds}{\int y^2 ds}$$

$$\mu = 25(30 - x)$$

$$OE = R\cos\theta$$

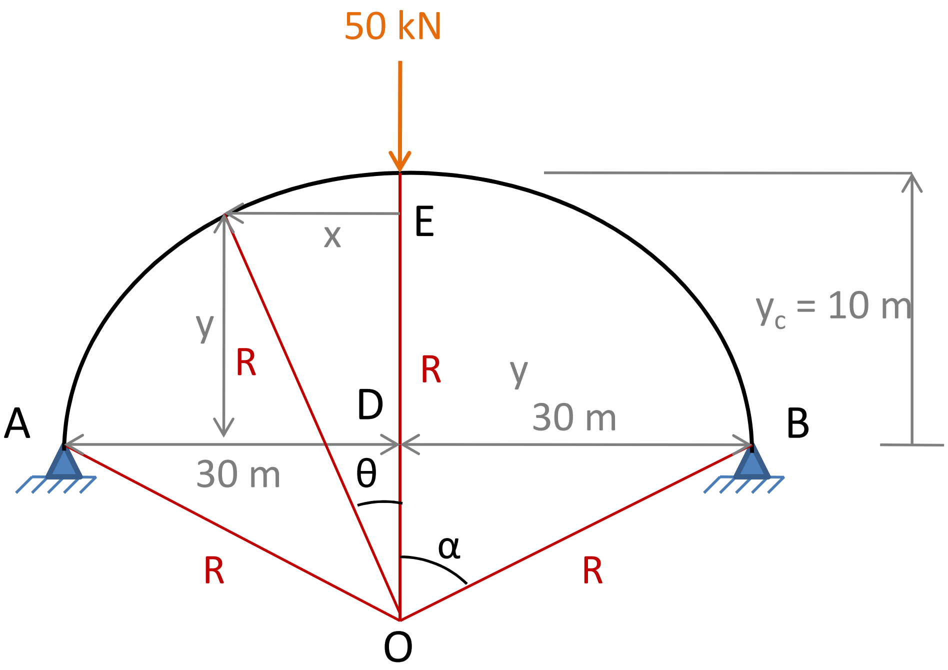
$$x = R\sin\theta$$

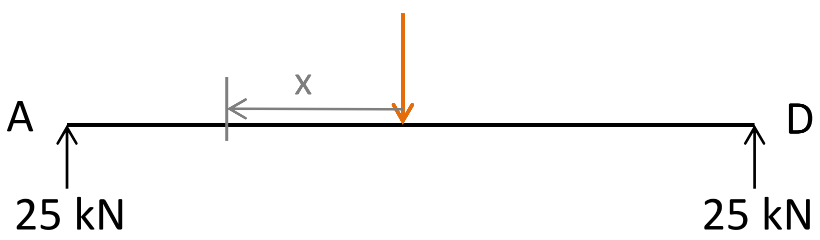
$$y = OE - OD$$

$$OD = 40\ m$$

$$y = R\cos\theta - 40$$

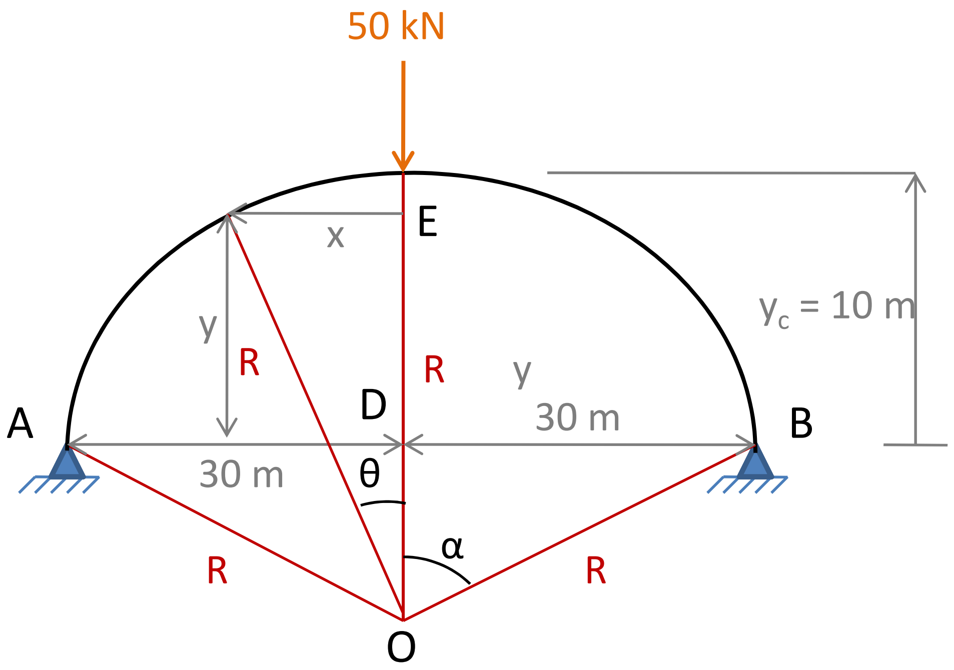
$$ds = Rd\theta$$





Solution

$$H = \frac{\int \mu y ds}{\int y^2 ds}$$

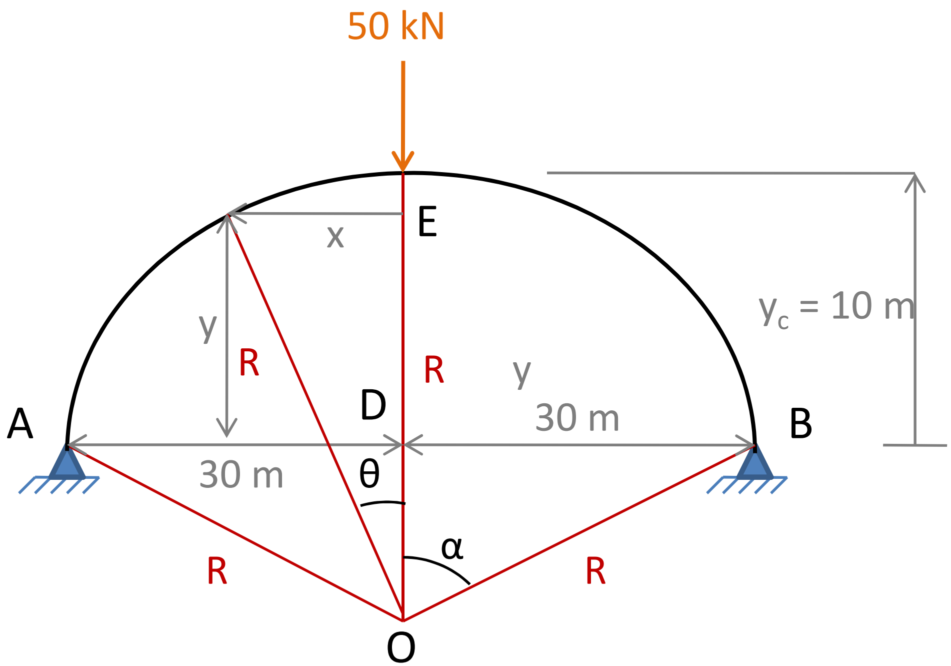


$$\int_0^L \mu y ds = 2\int_0^{0.6437} 25(30 - x)(R\cos\theta - 40)Rd\theta$$

$$= 2\int_0^{0.6437} 25(30 - R\sin\theta)(R\cos\theta - 40)Rd\theta = 194.5 \times 10^3\, kN - m^3$$

Solution

$$H = \frac{\int \mu y ds}{\int y^2 ds}$$



$$\int_0^L y^2 ds = 2\int_0^{0.6437} \left(R\cos\theta - 40\right)^2 R d\theta$$

$$= 3397.5\, m^3$$

$$H = \frac{194.5 \times 10^3}{3397.5} = 57.2\, kN$$

Thank You