Method of Consistent Deformation

Structural Analysis
By
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Theory of Structures-II

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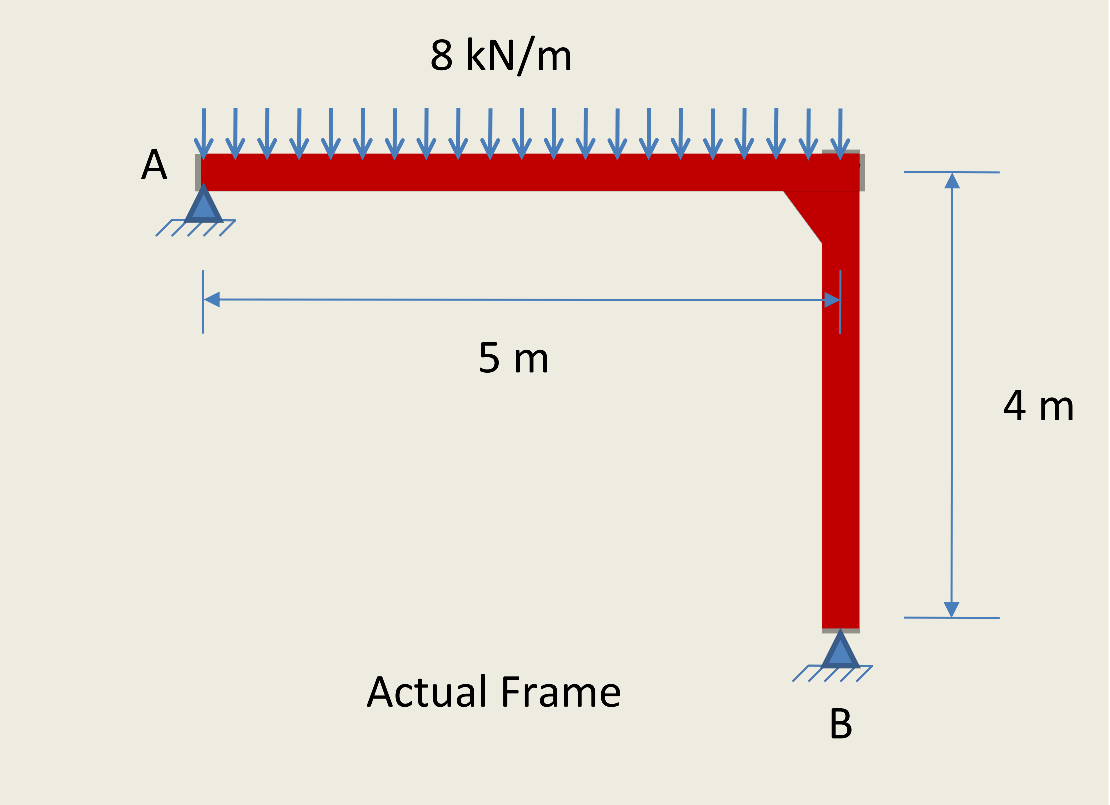
FRAMES

- Method of consistent deformation is very useful for solving problems involving statically indeterminate frames for single story and unusual geometry.

- Problems involving multistory frames, or with high indeterminacy are best solved using the slope deflection or moment distribution or the stiffness methods.

Example 6

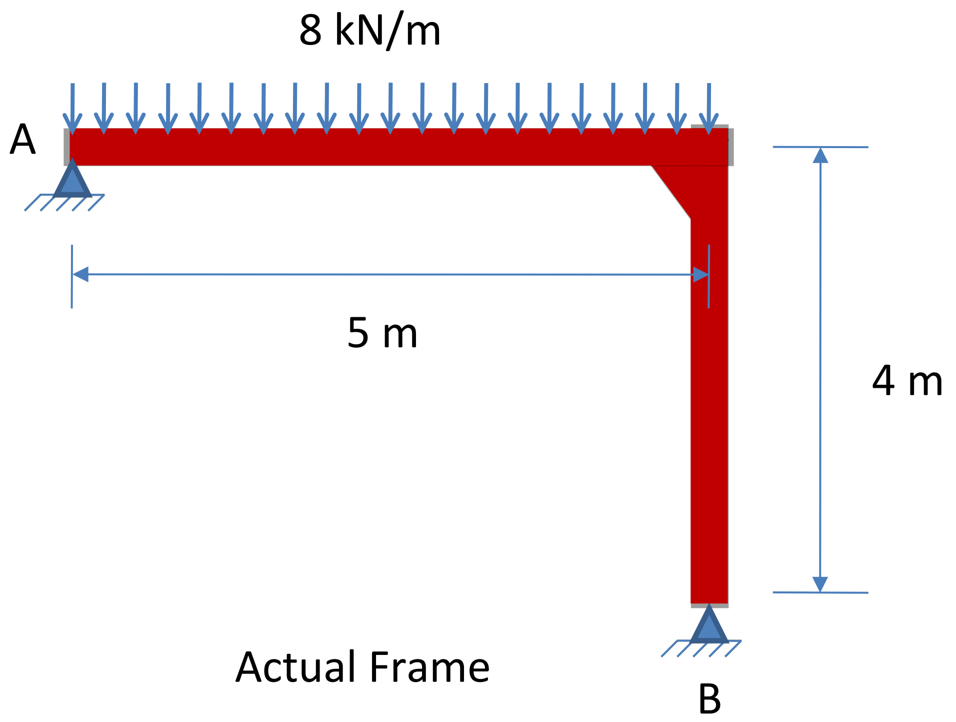
Determine the support reactions on the frame shown. EI is constant.



Actual Frame

Solution

Principle of Superposition



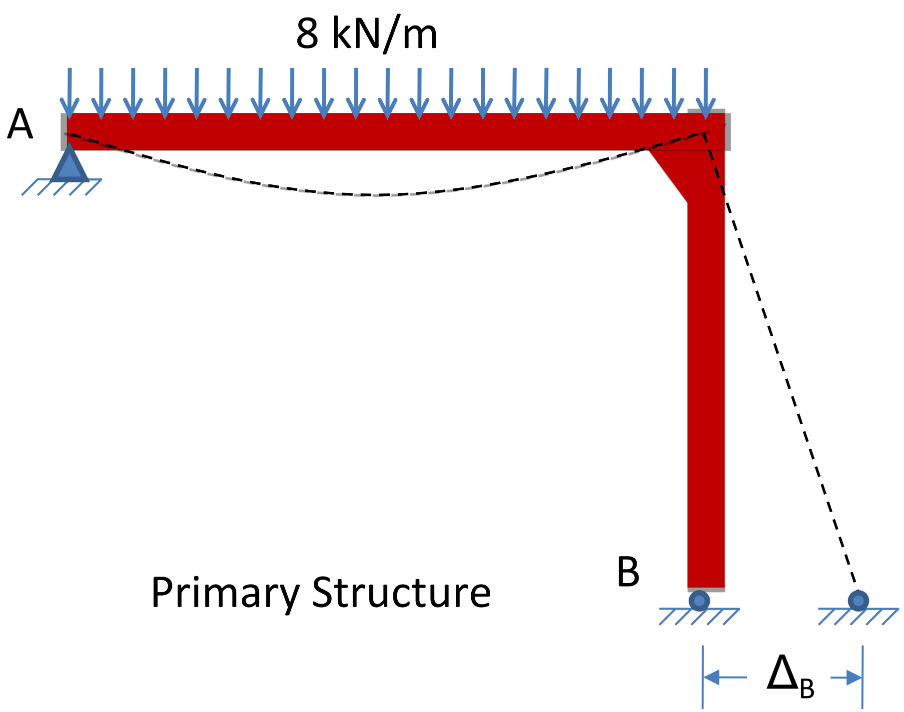
Actual Frame

- By inspection the frame is indeterminate to the first degree.

Solution

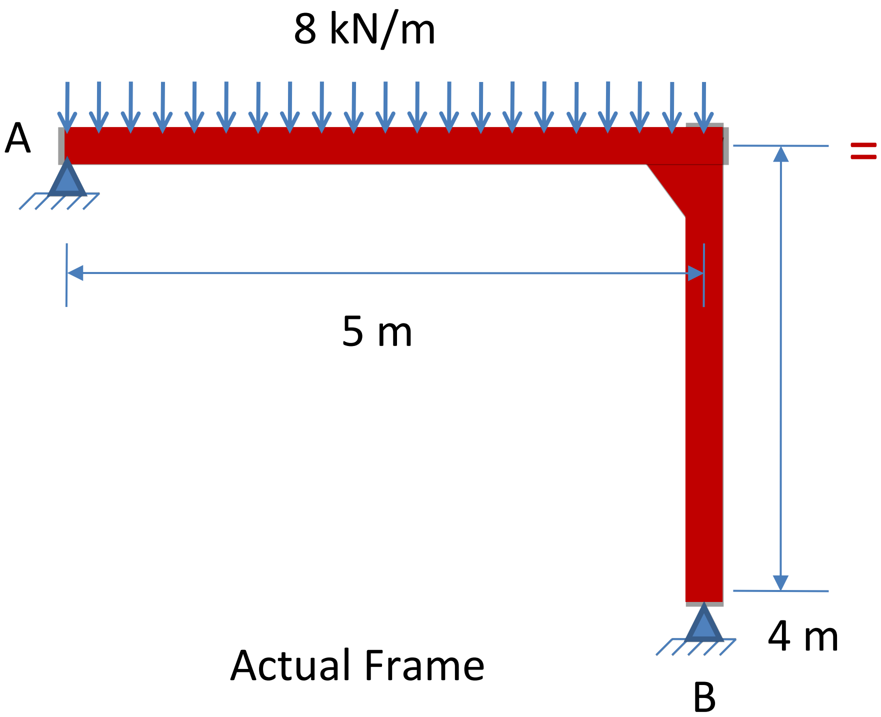
Principle of Superposition

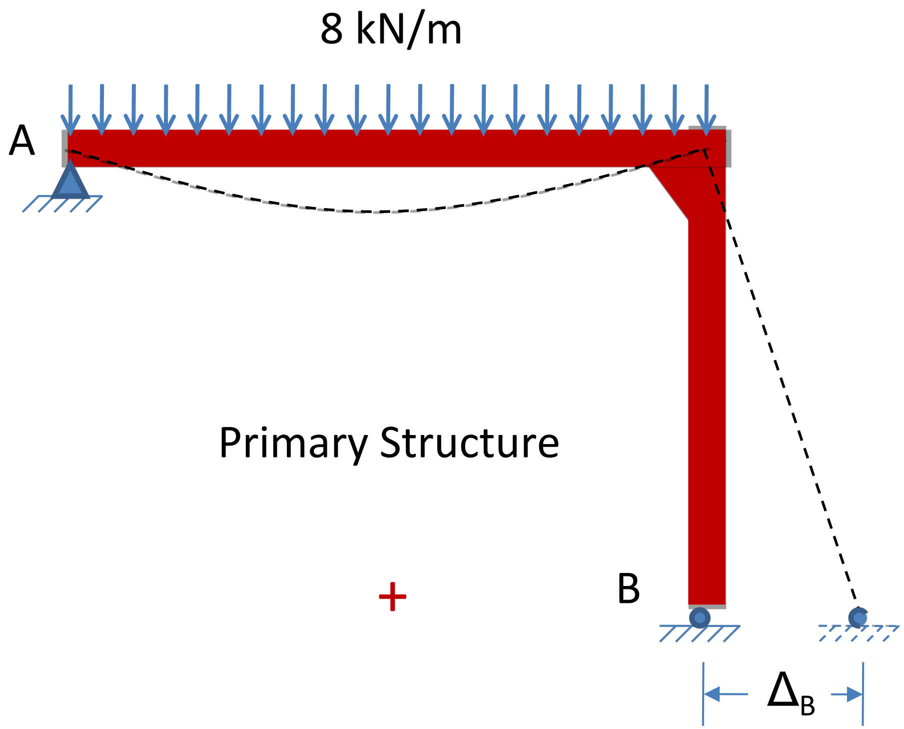
- We will choose the horizontal reaction at support B as the redundant.
- The pin at B is replaced by the roller, since a roller will not constraint B in the horizontal direction.

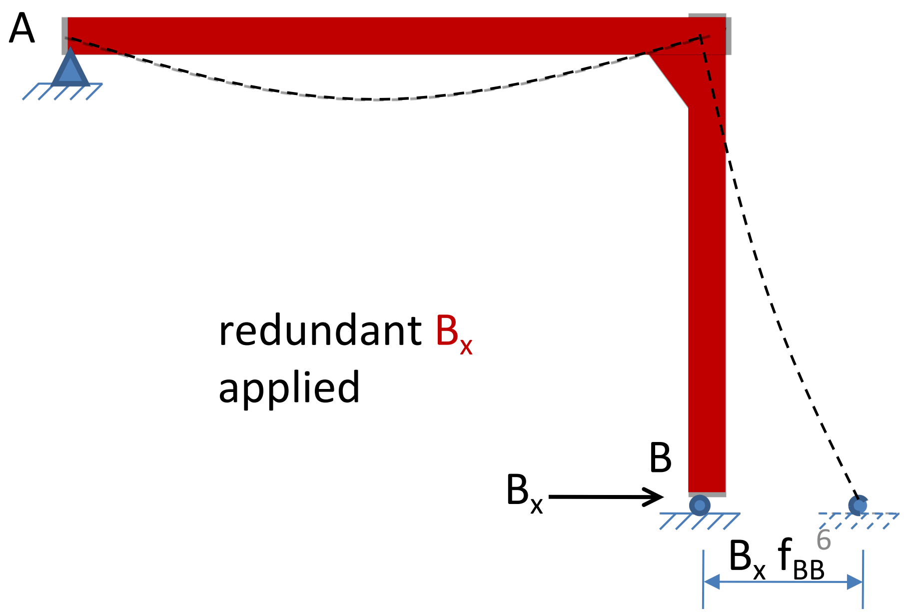


Solution

Principle of Superposition



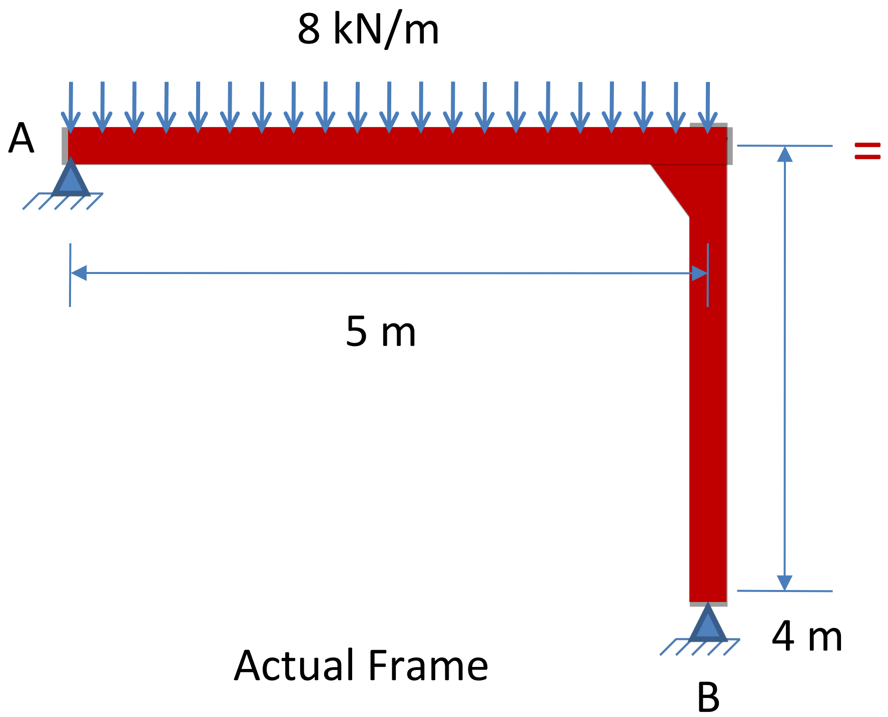




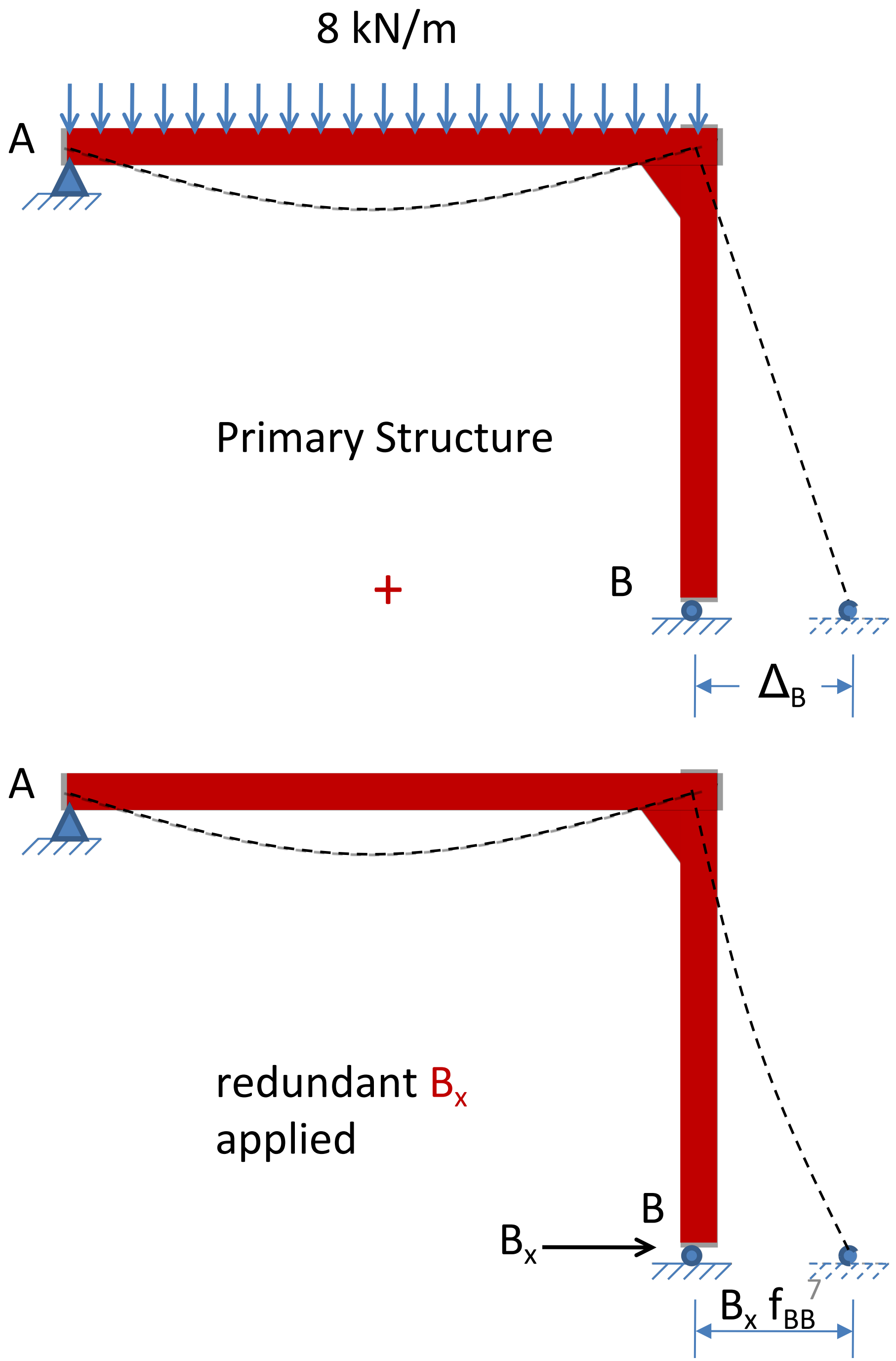
Solution

Compatibility Equation

$$\left(\overset{+}{\rightarrow}\right) \qquad 0 = \Delta_B + B_x f_{BB}$$



Actual Frame



Solution

Compatibility Equation

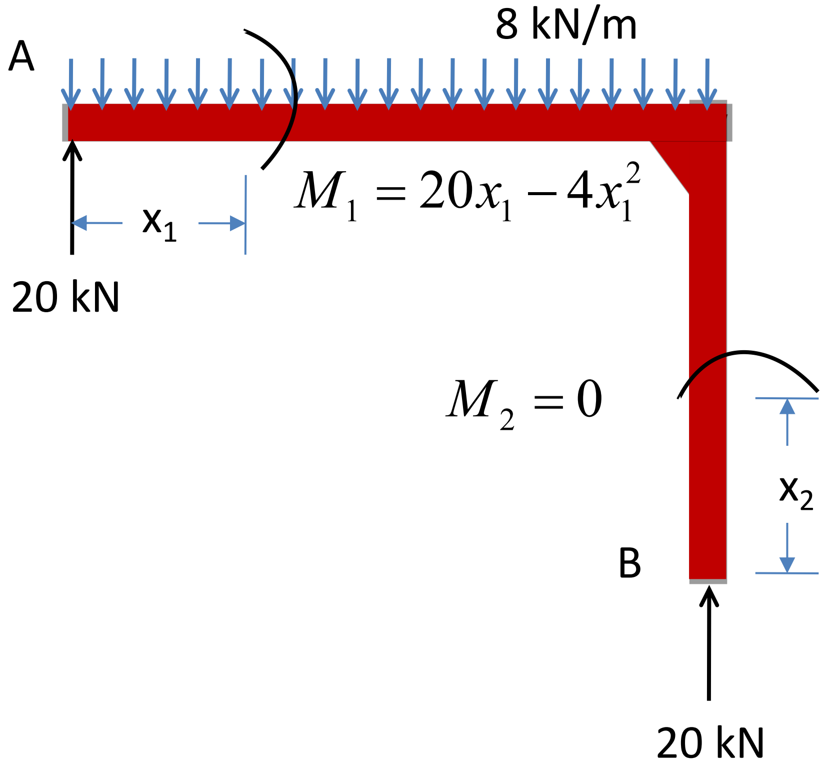
$$0 = \Delta_B + B_x f_{BB}$$

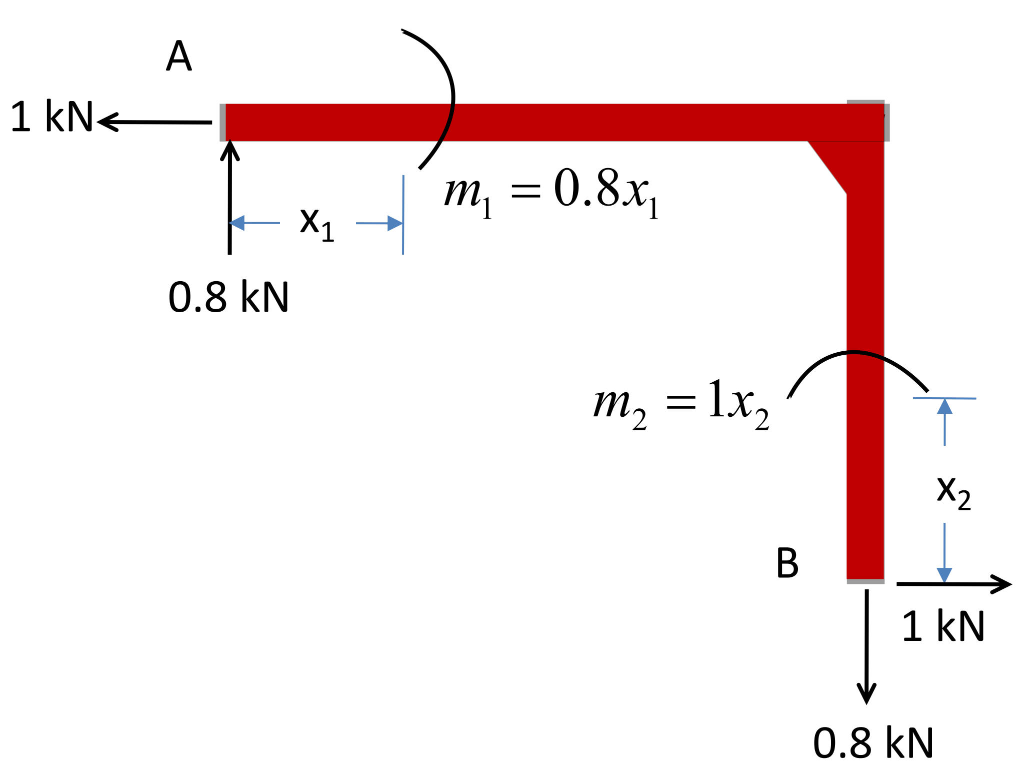
- The terms Δ_B and f_{BB} will be computed using the method of virtual work.
- The frame's x coordinates and internal moments are shown in figure.
- It is important that in each case the selected coordinate x_1 or x_2 be the same for both the real and virtual loadings.
- Also the positive directions for M and m must be same.

Solution

Compatibility Equation

- For Δ_B we require application of real loads and a virtual unit load at B

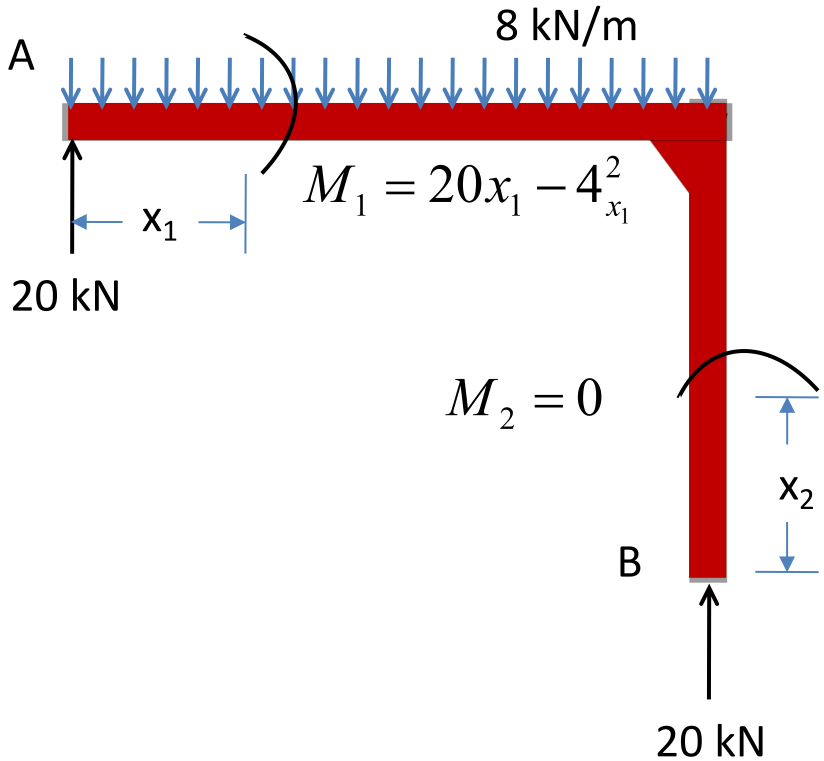


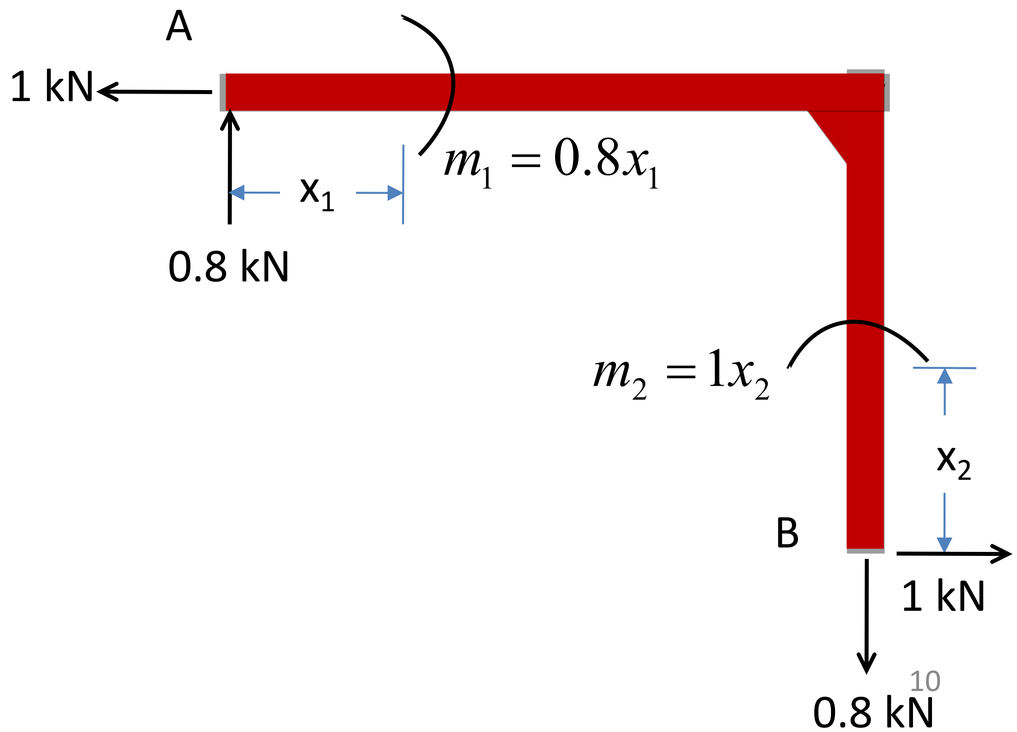


Solution

$$\Delta_B = \int_0^L \frac{Mm}{EI}dx = \int_0^5 \frac{(20x_1 - 4x_1^2)(0.8x_1)dx_1}{EI} + \int_0^4 \frac{0(1x_2)dx_2}{EI}dx$$

$$= \frac{166.7}{EI} + 0 = \frac{166.7}{EI}$$

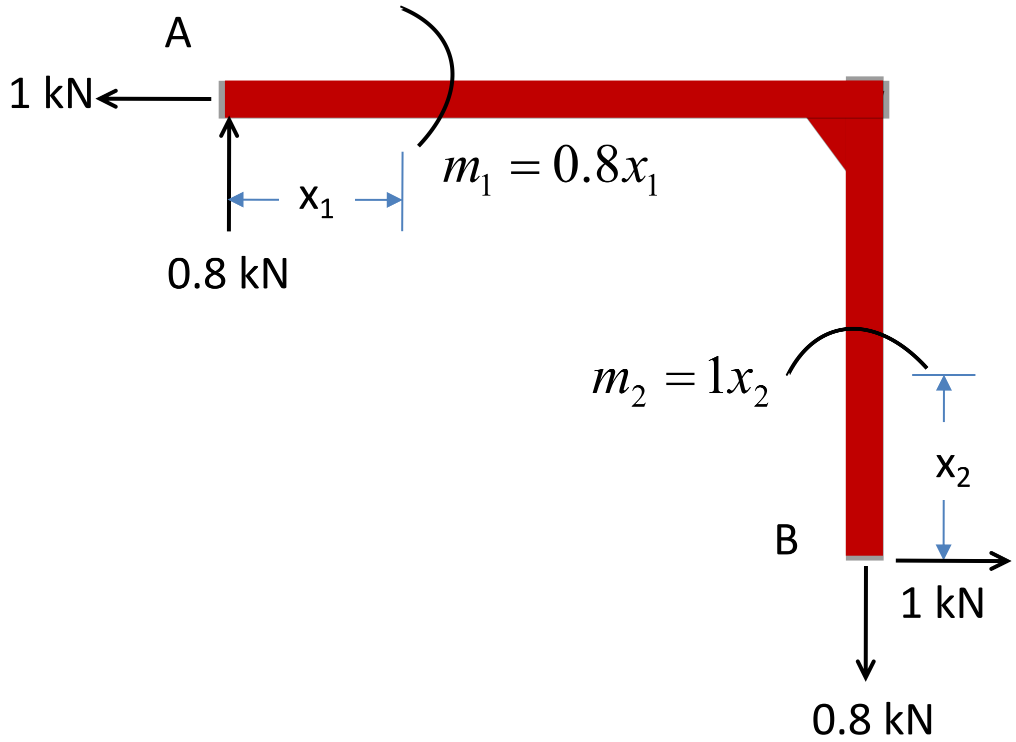




Solution

Compatibility Equation

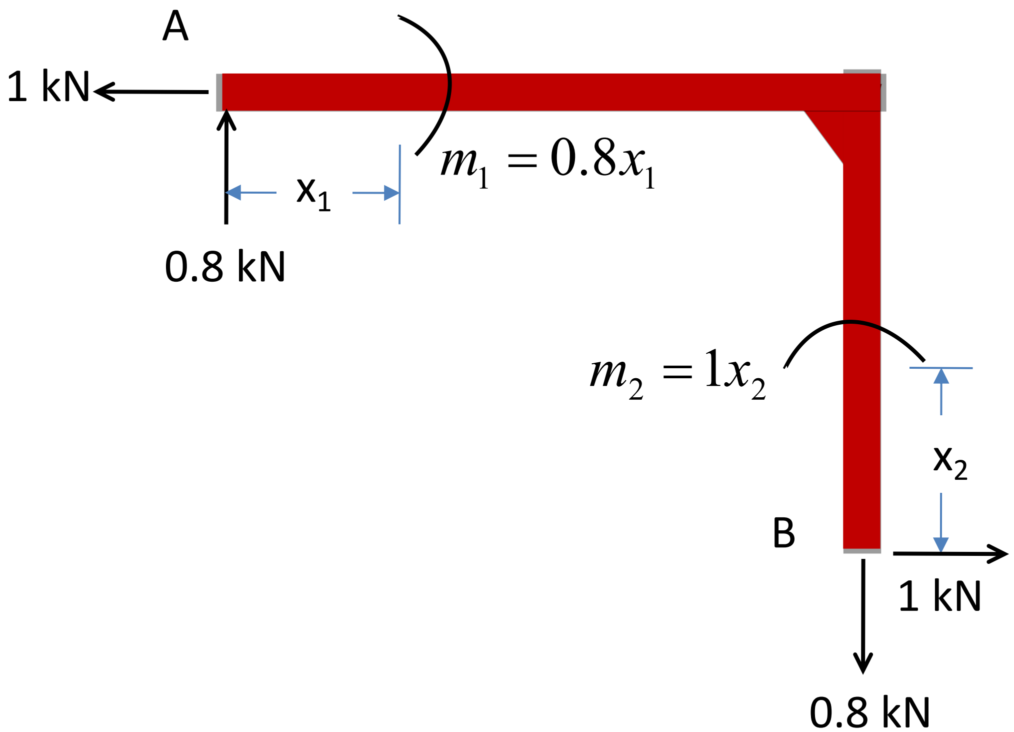
- For f_{BB} we require application of real unit load acting at B and a virtual unit load acting at B



Solution

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^5 \frac{(0.8x_1)^2\,dx_1}{EI} + \int_0^4 \frac{0(1x_2)^2\,dx_2}{EI}dx$$

$$= \frac{26.7}{EI} + \frac{21.3}{EI} = \frac{48.0}{EI}$$



Solution

Compatibility Equation

$$0 = \Delta_B + B_x f_{BB} \qquad (1)$$

- Substituting the data in Eq. (1)

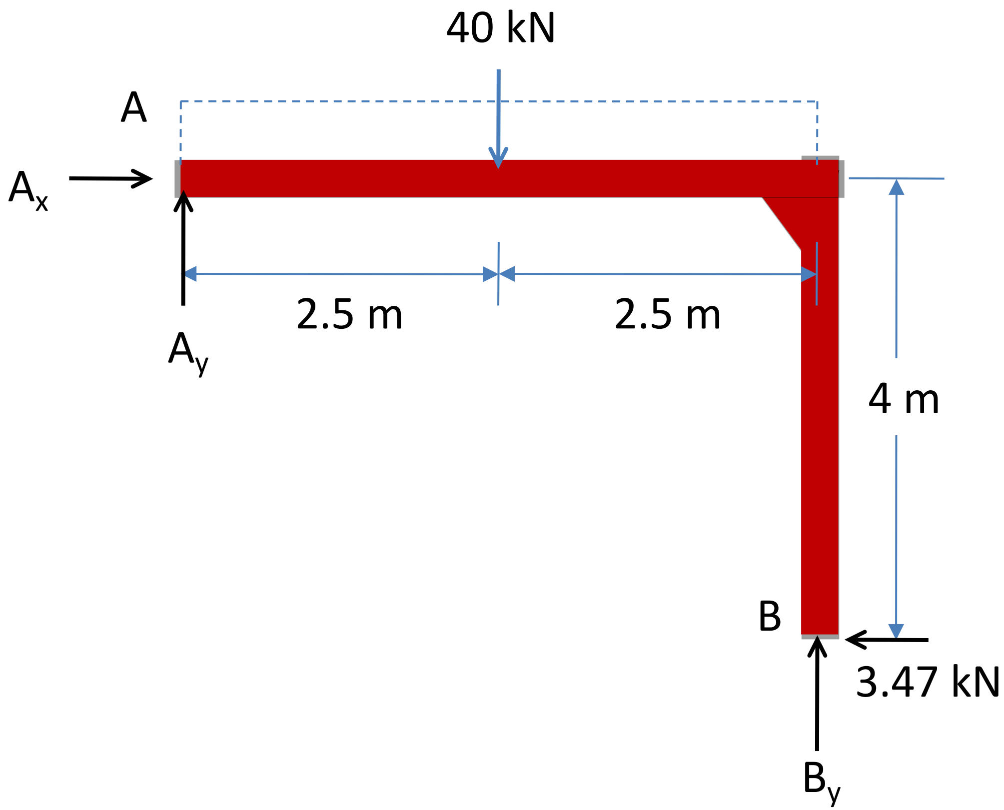
$$0 = \frac{166.7}{EI} + B_x\left(\frac{48.0}{EI}\right)$$

$$B_x = -3.47 \text{ kN} \qquad \text{ANS}$$

Solution

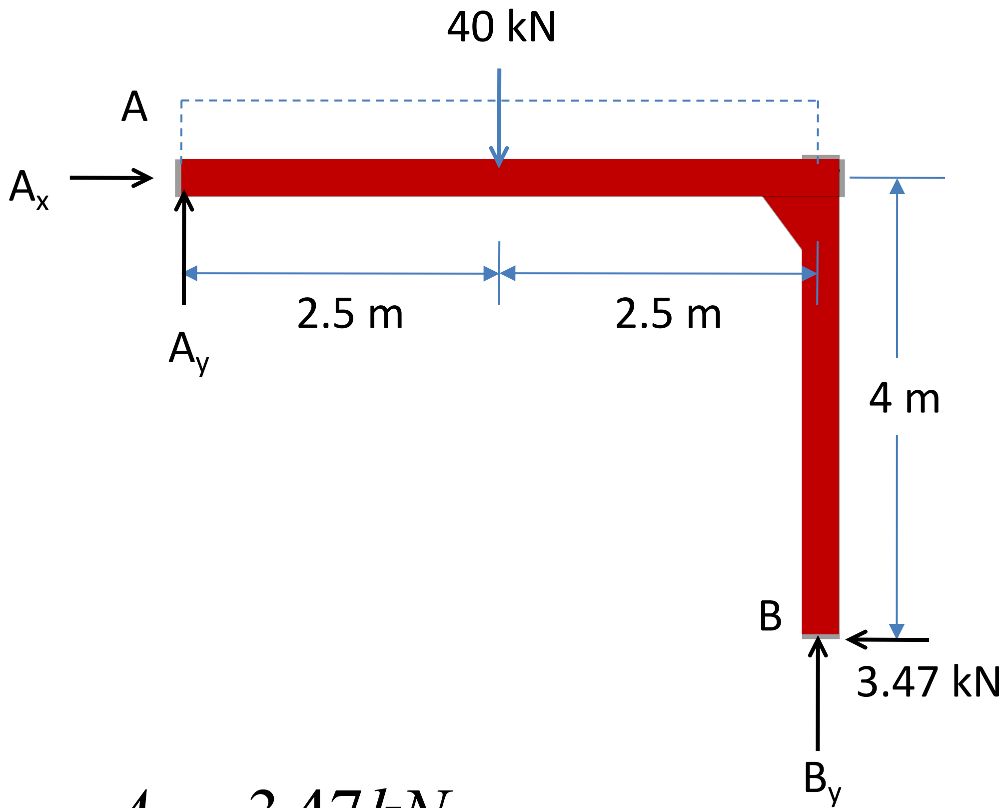
Equilibrium Condition

- Showing B_x on the free body diagram of the frame in the correct direction, and applying the equations of equilibrium, we have



Solution

Equilibrium Condition



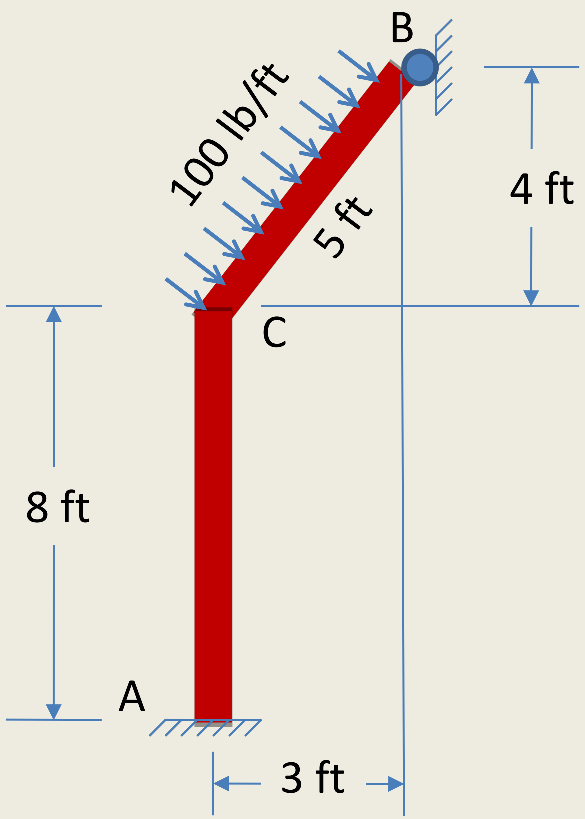
$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad A_x - 3.47 = 0 \qquad A_x = 3.47 kN$$

$$(+\sum M_A = 0; \quad -40(2.5) + B_y(5) - 3.47(4) = 0 \quad B_y = 22.8\ kN$$

$$+\uparrow\sum F_y = 0; \qquad A_y - 40 + 22.8 = 0 \qquad A_y = 17.2\ kN$$

Example 7

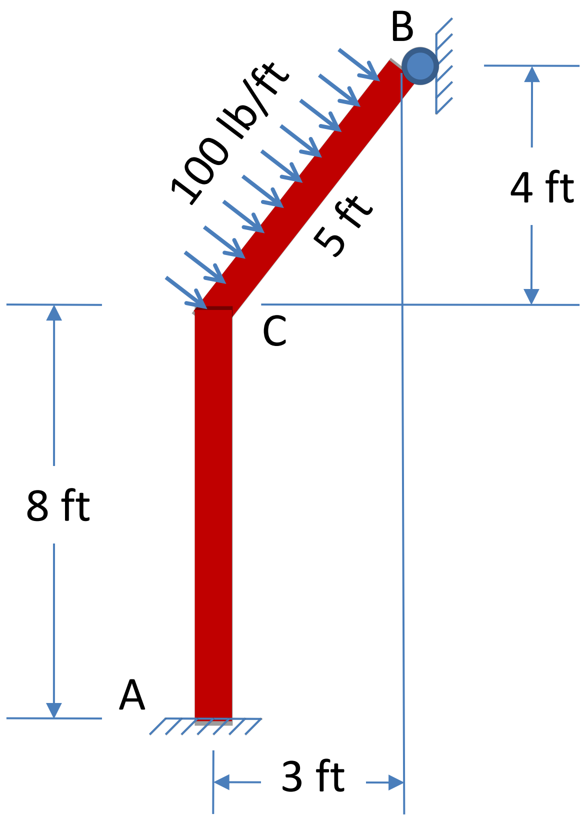
Determine the moment at fixed support A for the frame shown. EI is constant.



Actual Frame

Solution

Principle of Superposition



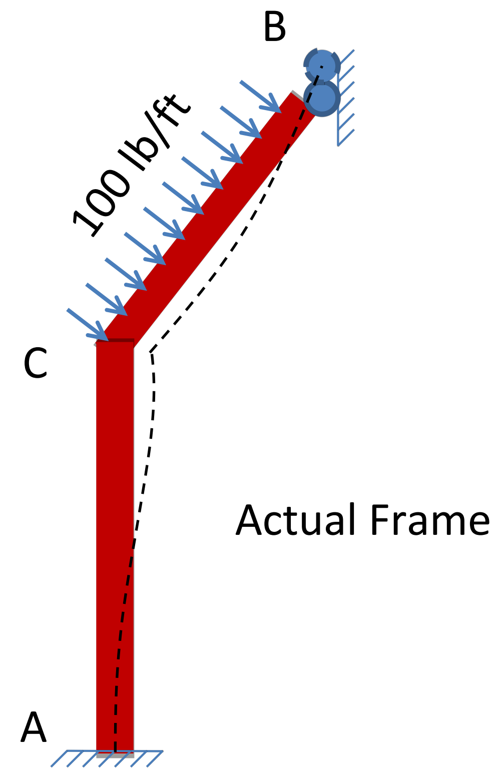
Actual Frame

- By inspection the frame is indeterminate to the first degree.

Solution

Principle of Superposition

- M_A can be directly obtained by choosing as the redundant.
- The capacity of the frame to support a moment at A is removed and therefore a pin is used at A for support.

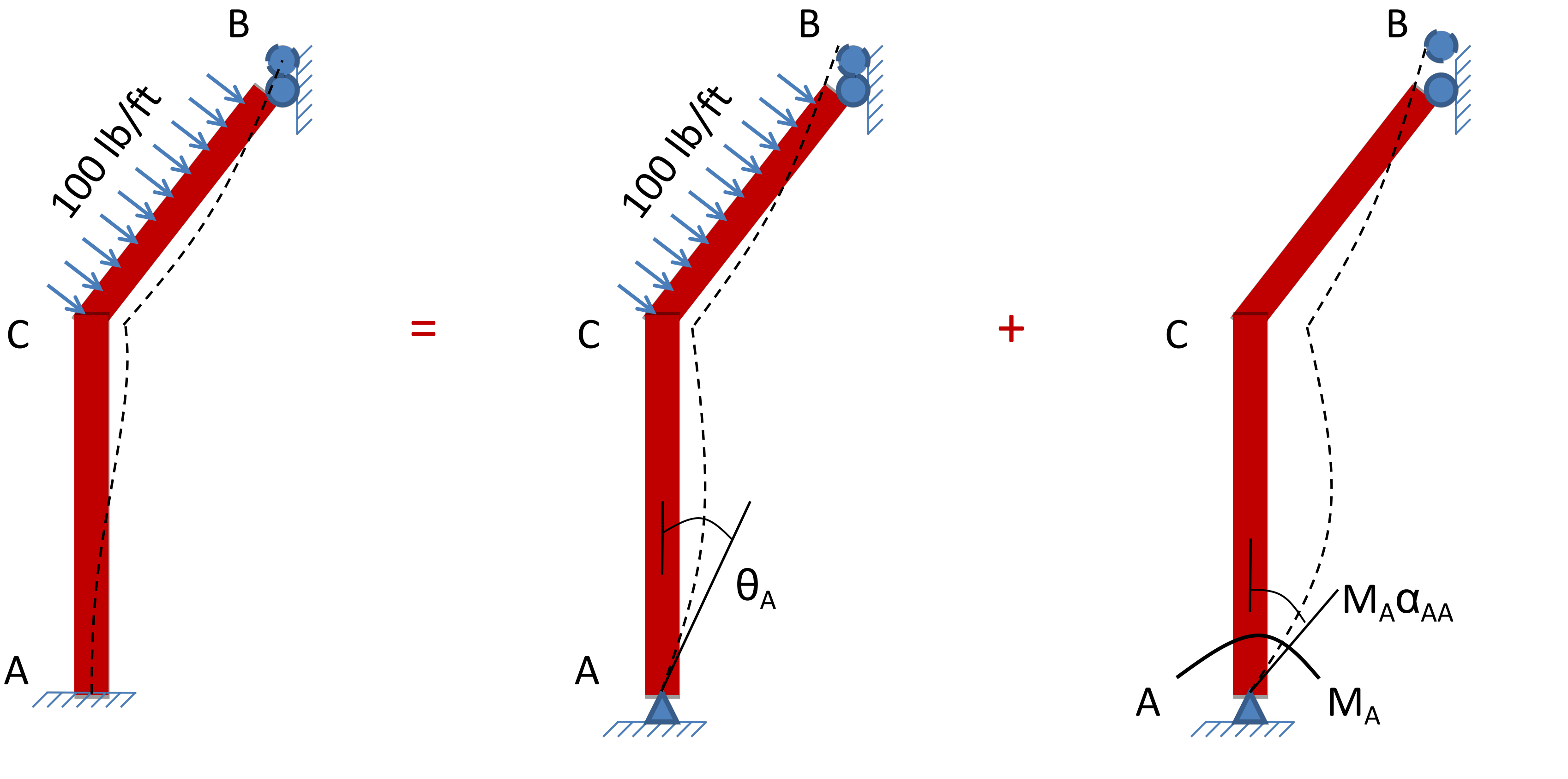


Solution

Compatibility Equation

$$(\circlearrowleft +) \qquad 0 = \theta_A + M_A \alpha_{AA} \qquad (1)$$

Reference to point A



actual frame

primary structure

Redundant M_A applied

Solution

Compatibility Equation

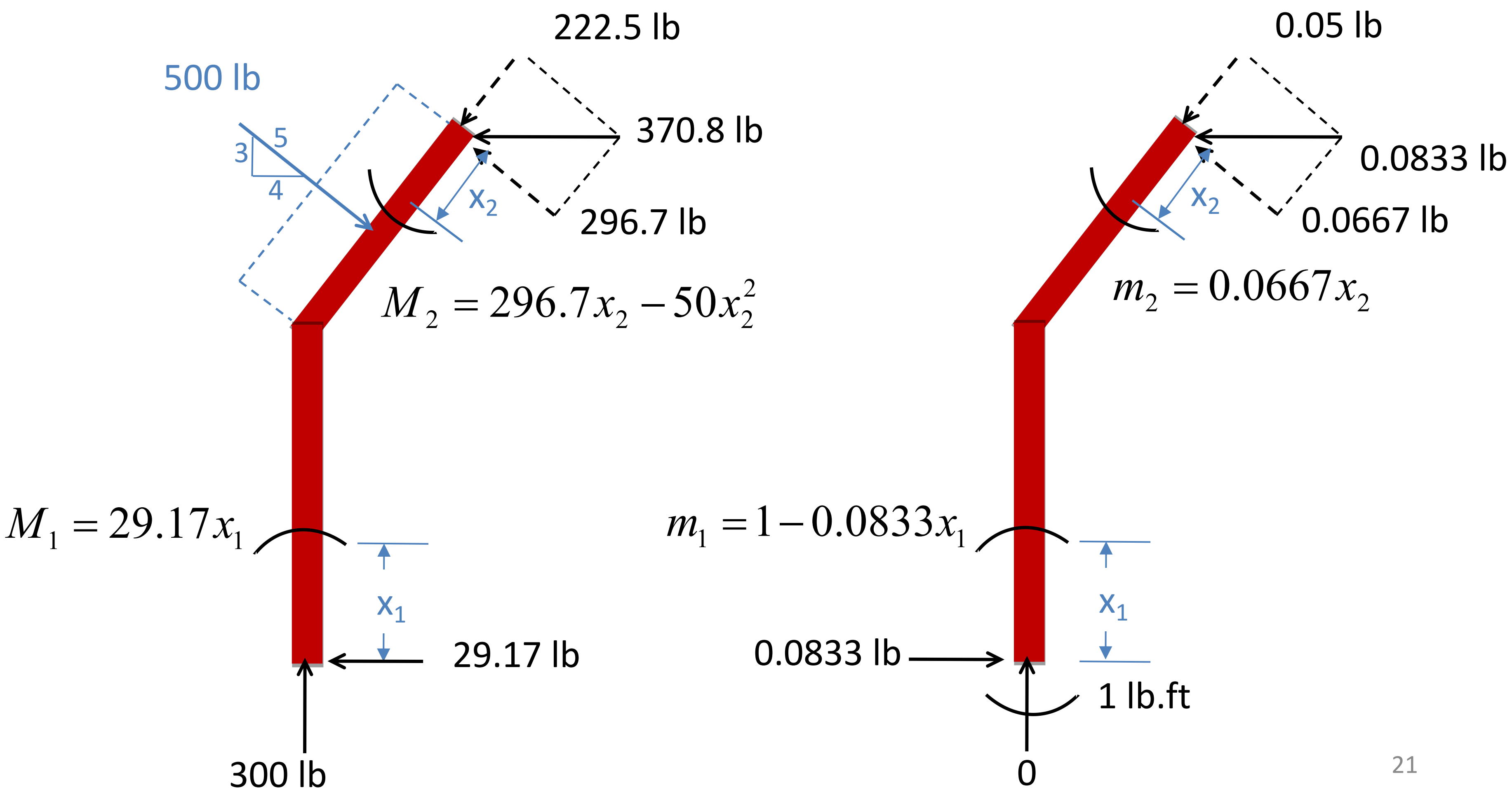
$$(\curvearrowleft +) \qquad 0 = \theta_A + M_A \alpha_{AA} \qquad (1)$$

- The terms θ_A and α_{AA} will be computed using the method of virtual work.
- The frame ' s x coordinates and internal moments are shown in figure.

Solution

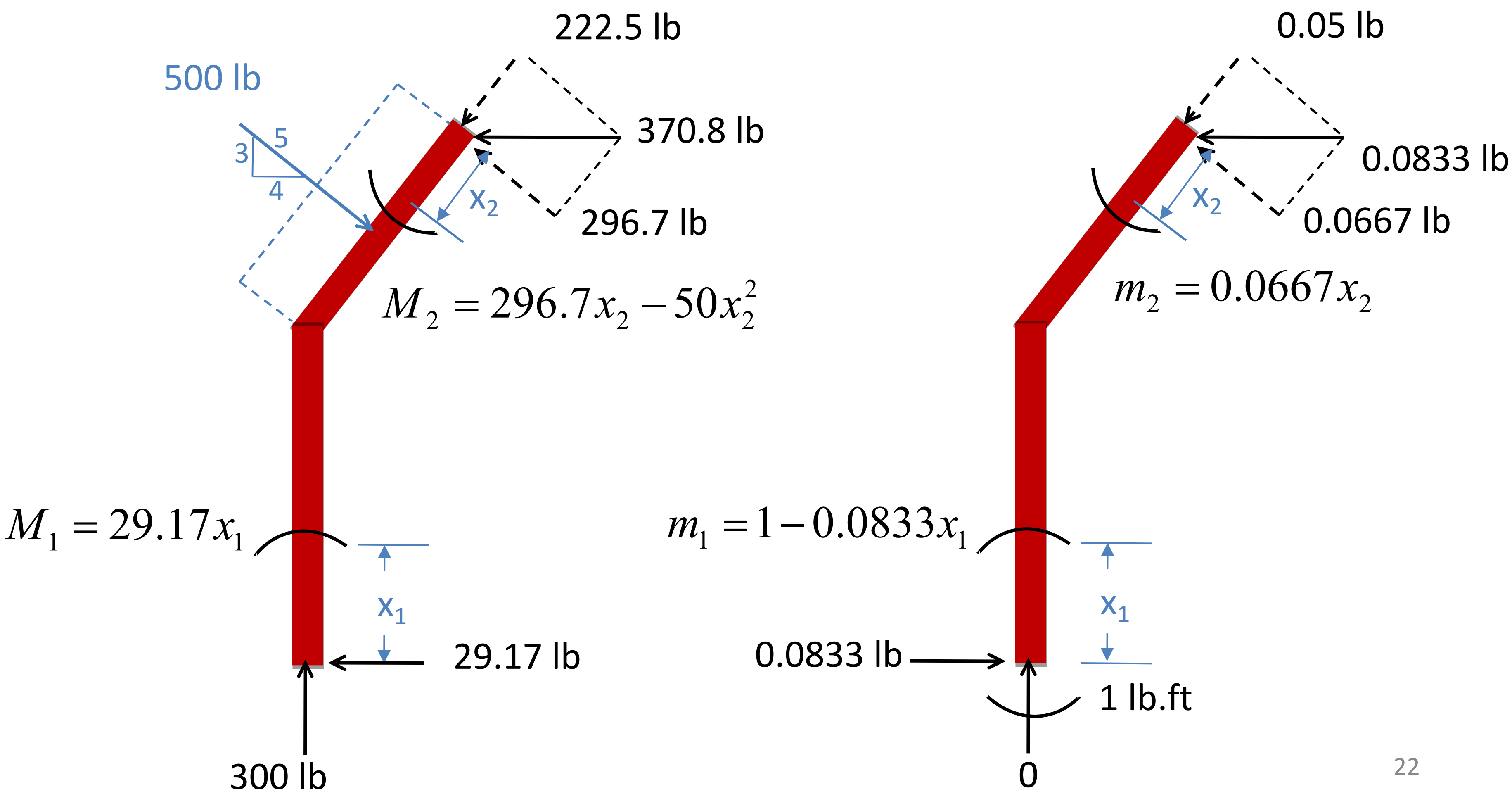
Compatibility Equation

Reference to point A



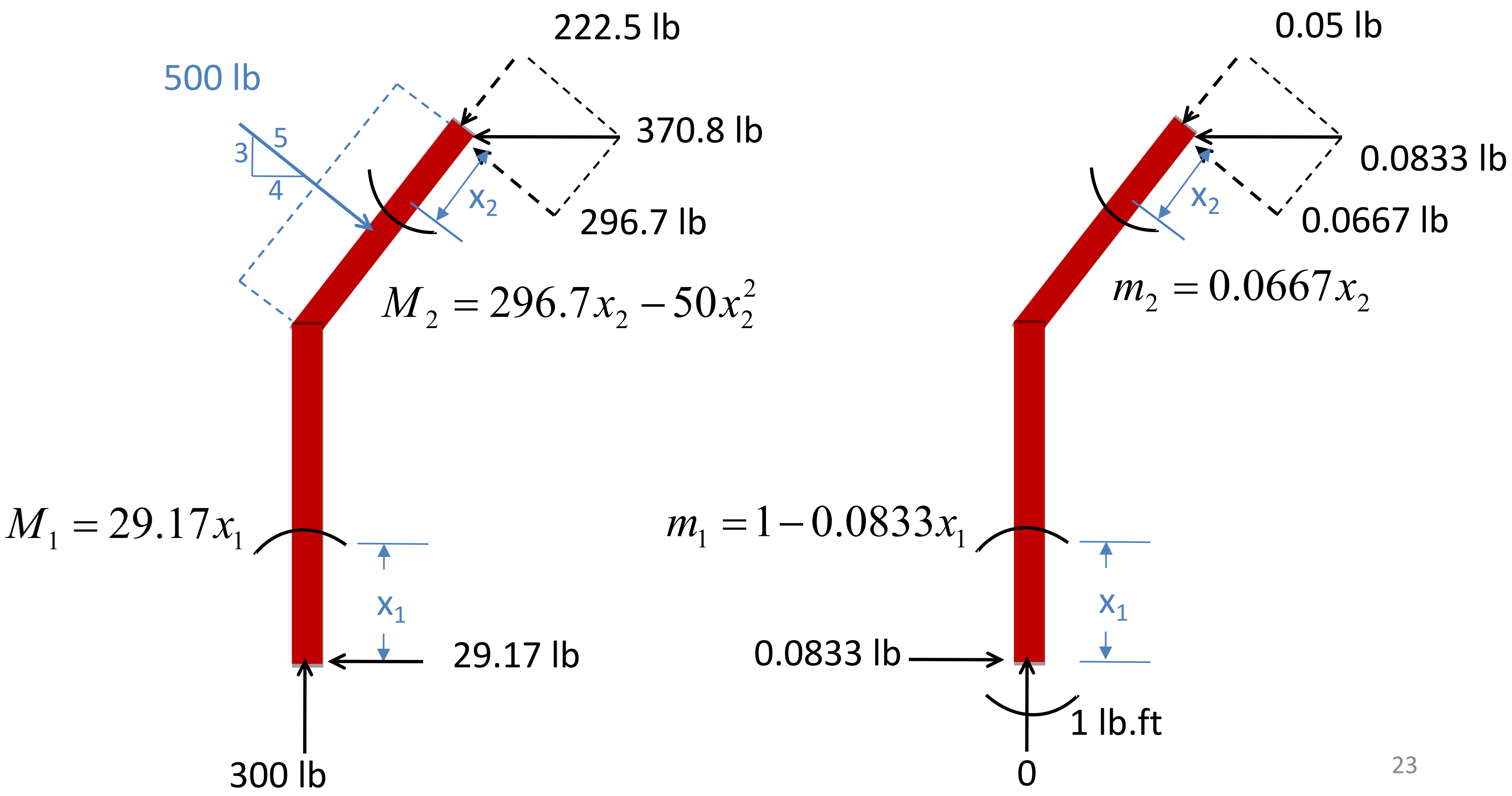
Solution

For θ_A we require application of real loads and a virtual unit couple moment at A



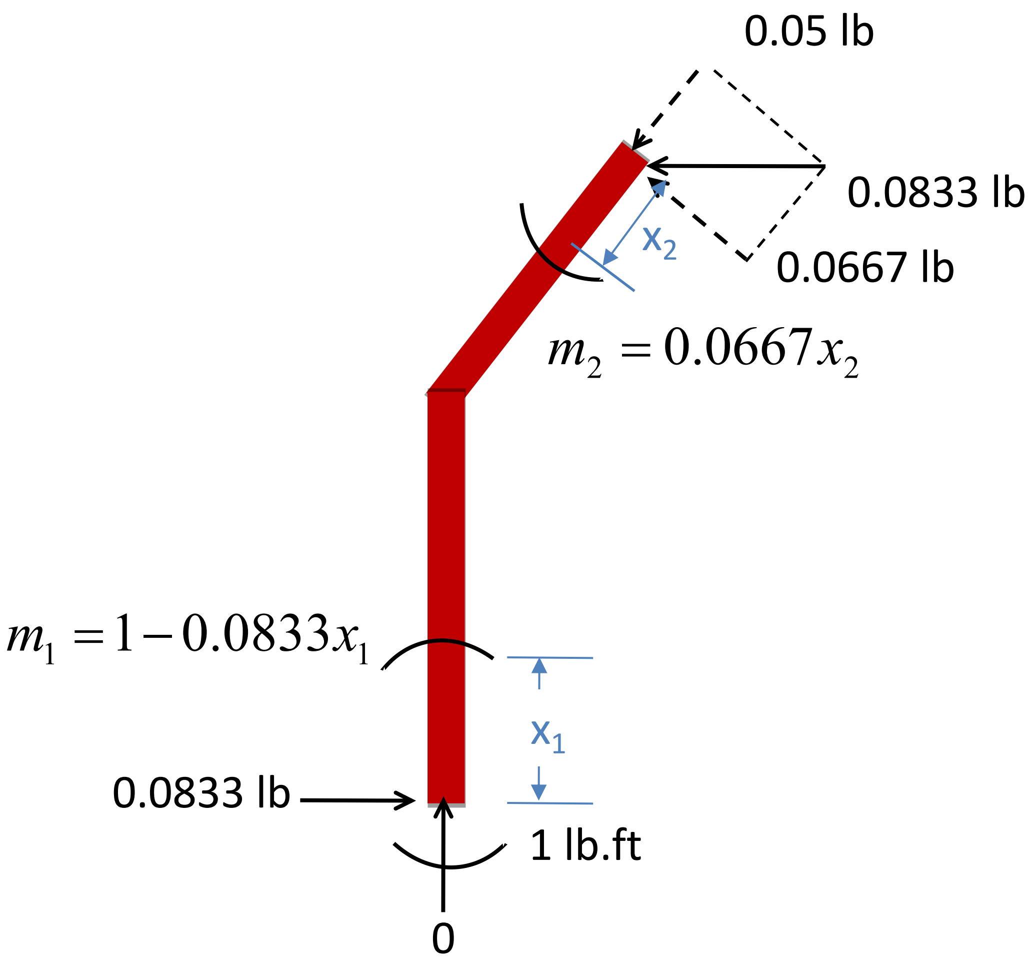
$$\theta_A = \sum \int_0^L \frac{Mm_\theta}{EI} dx = \int_0^8 \frac{(29.17x_1)(1-0.0833x_1)dx_1}{EI} + \int_0^5 \frac{(296.7x_2 - 50x_2^2)(0.0067x_2)dx_2}{EI}$$

$$= \frac{518.5}{EI} + \frac{303.2}{EI} = \frac{821.8}{EI}$$



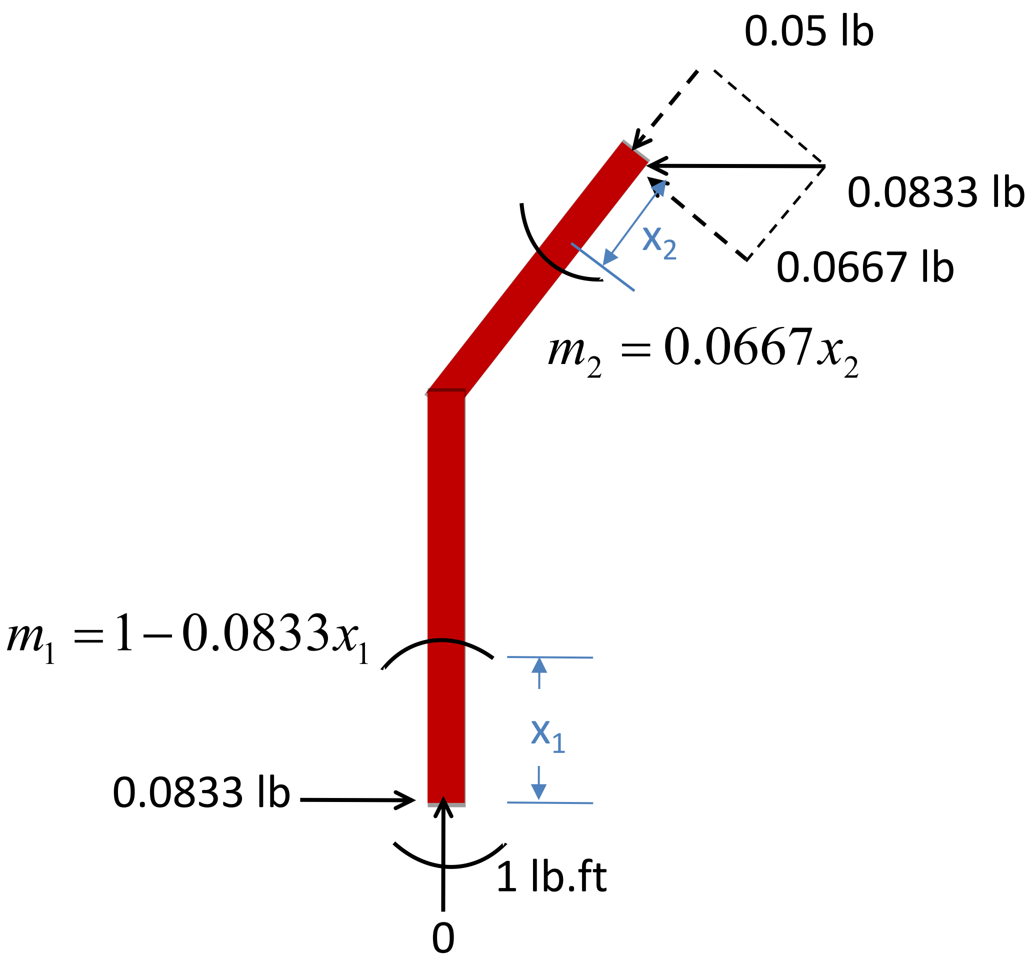
Solution

For α_{AA} we require application of real unit couple moment and a virtual unit couple moment at A



$$\alpha_{AA} = \sum \int_0^L \frac{m_\theta m_\theta}{EI} dx = \int_0^8 \frac{(1 - 0.0833x_1)^2 dx_1}{EI} + \int_0^5 \frac{(0.0067x_2)^2 dx_2}{EI}$$

$$= \frac{3.85}{EI} + \frac{0.185}{EI} = \frac{4.04}{EI}$$

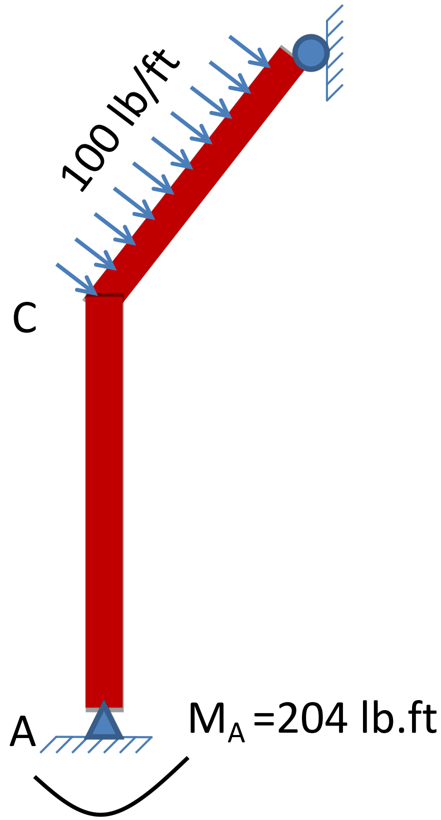


Solution

Substituting these results into Eq. (1), and solving yields

$$0 = \frac{821.8}{EI} + M_A\left(\frac{4.04}{EI}\right)$$

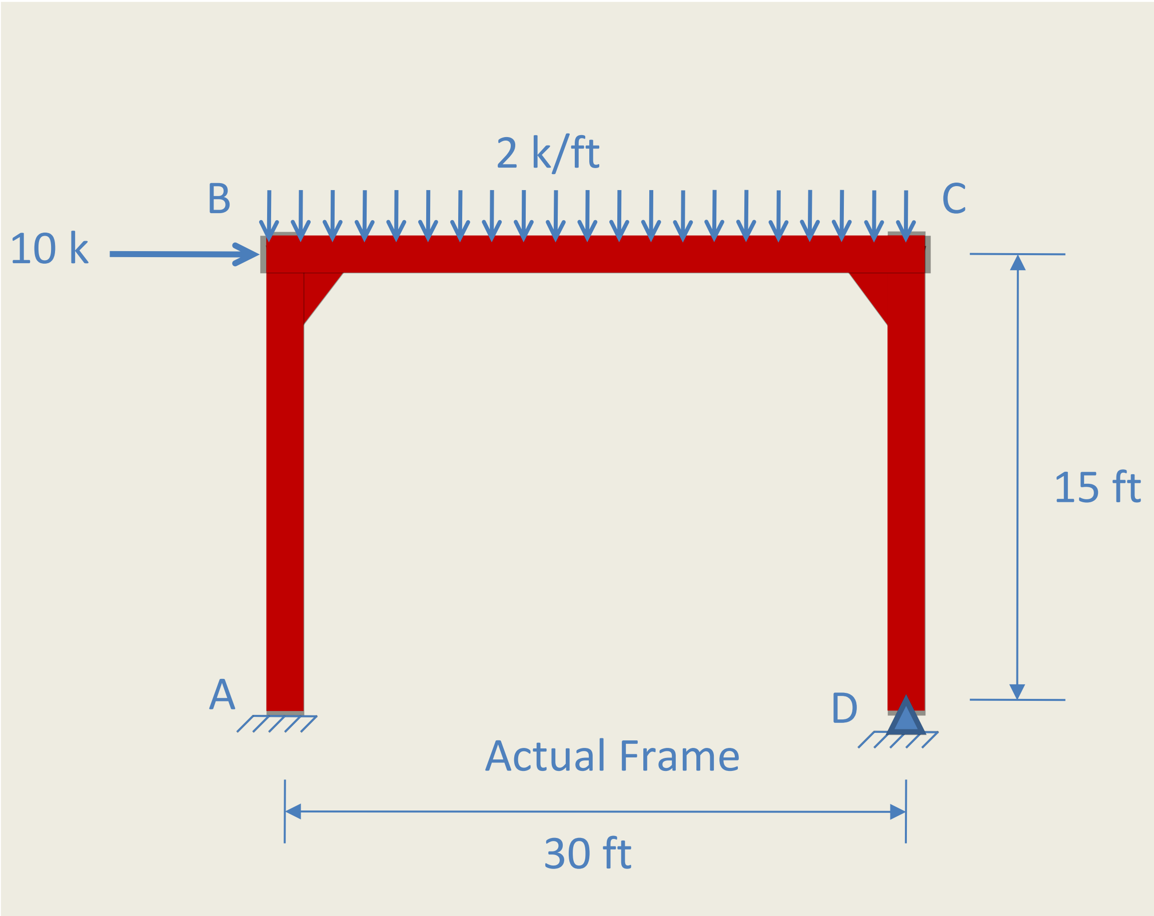
$$M_A = -204\, lb.ft \qquad \text{ANS}$$



The negative sign indicates M_A acts in opposite direction to that shown in figure.

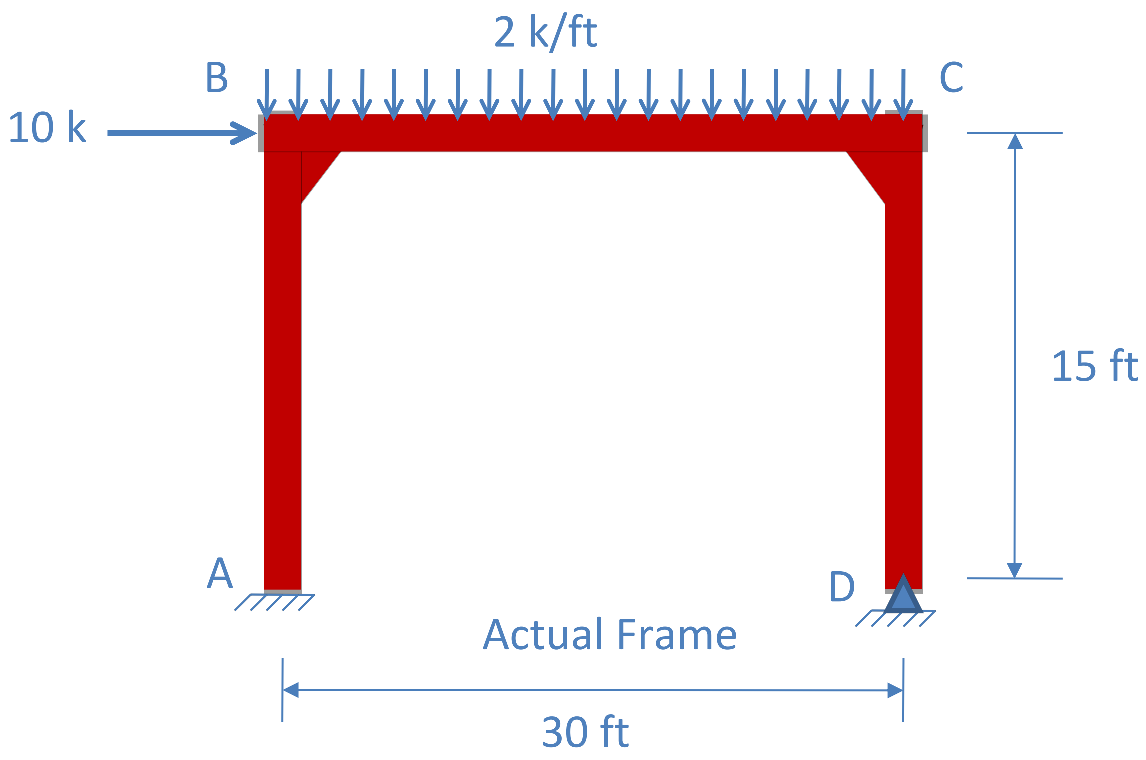
Example 8

Determine the reactions and draw the shear and bending moment diagrams. EI is constant.



Solution

Principle of Superposition



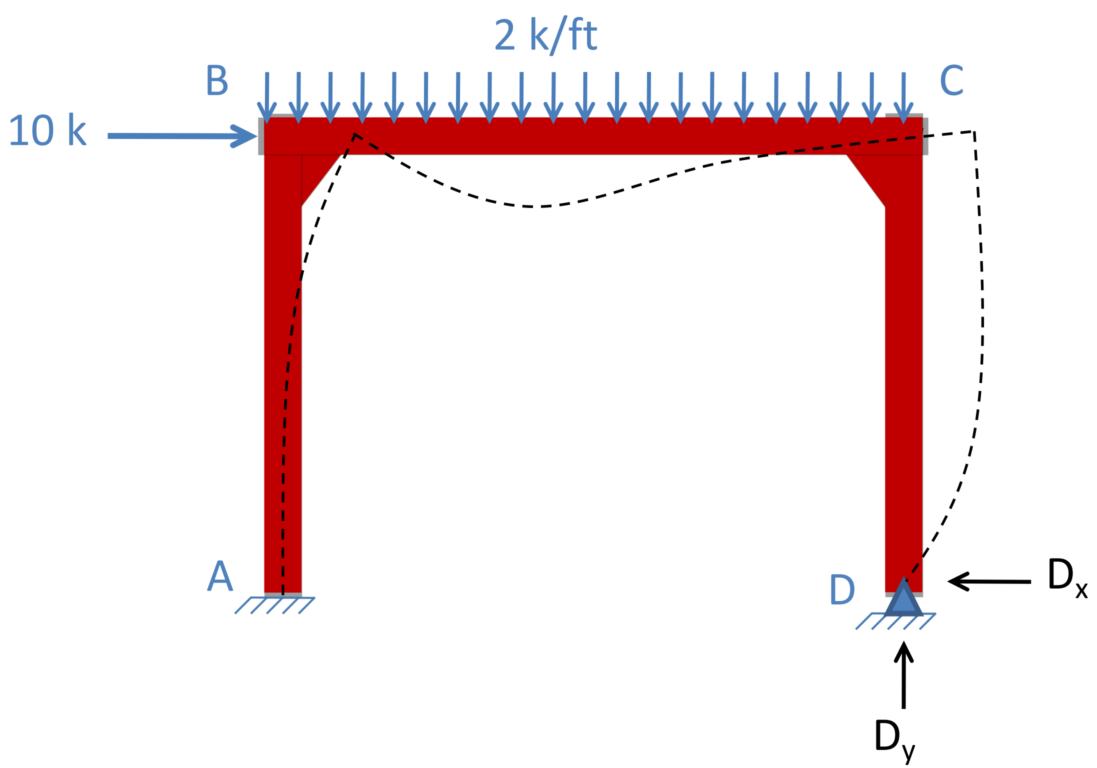
- Degree of indeterminacy = 2

Solution

Principle of Superposition

- We will choose the horizontal reaction D_x and vertical reaction D_y at point D as the redundants.

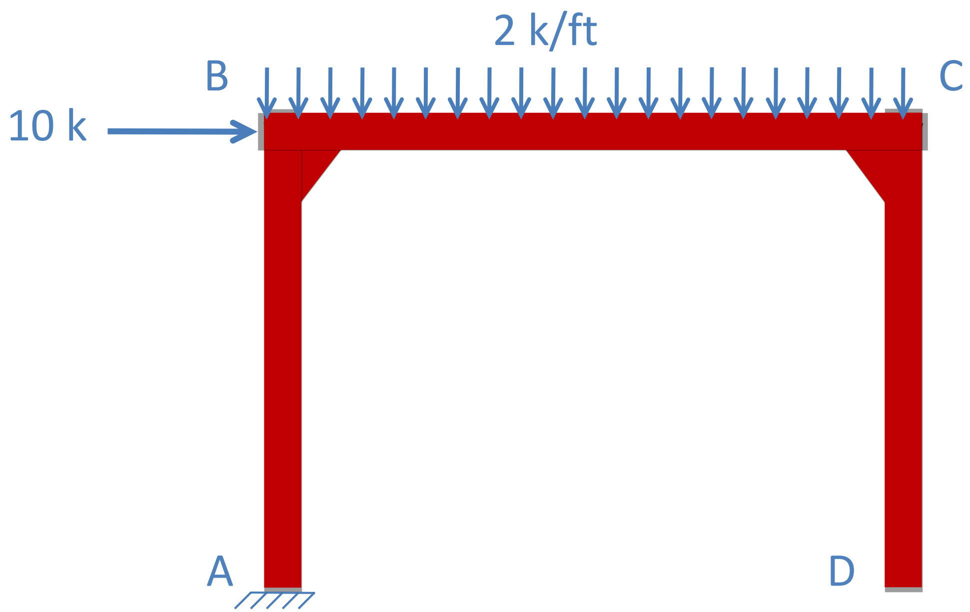
Actual Frame



Solution

Principle of Superposition

- Primary structure is obtained by removing the hinged support at point D.

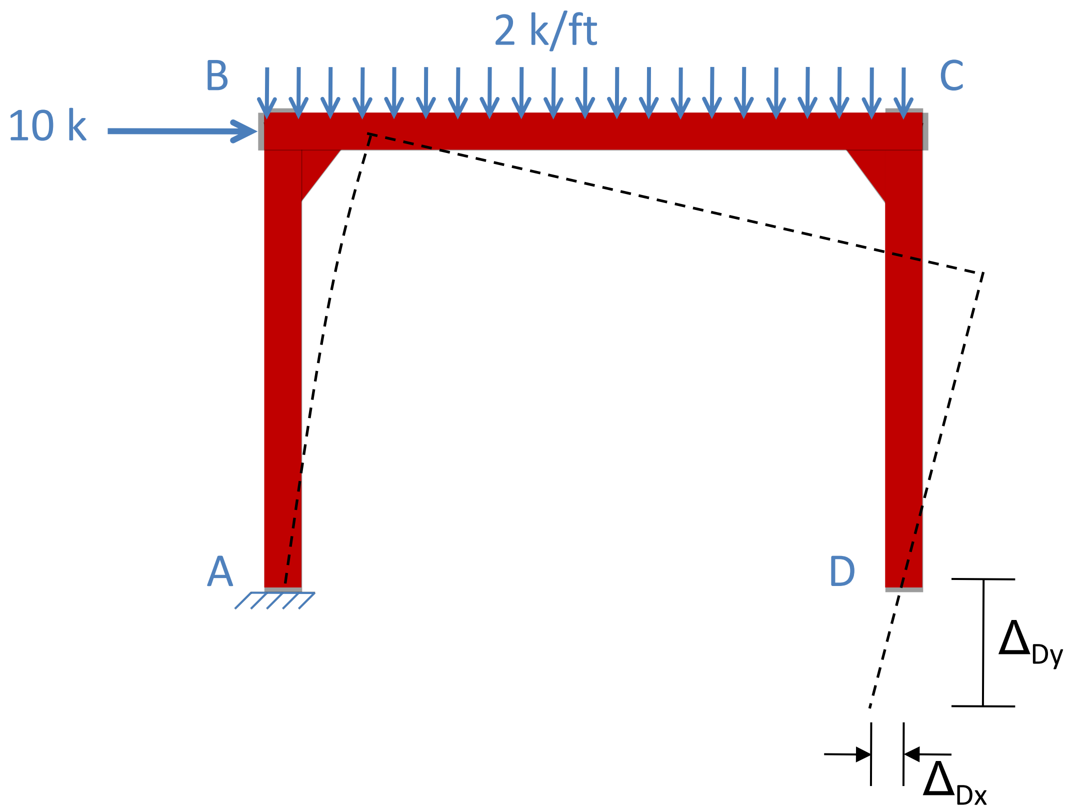


Primary Structure

Solution

Principle of Superposition

- Primary structure is subjected separately to the external loading and redundants D_x and D_y as shown.

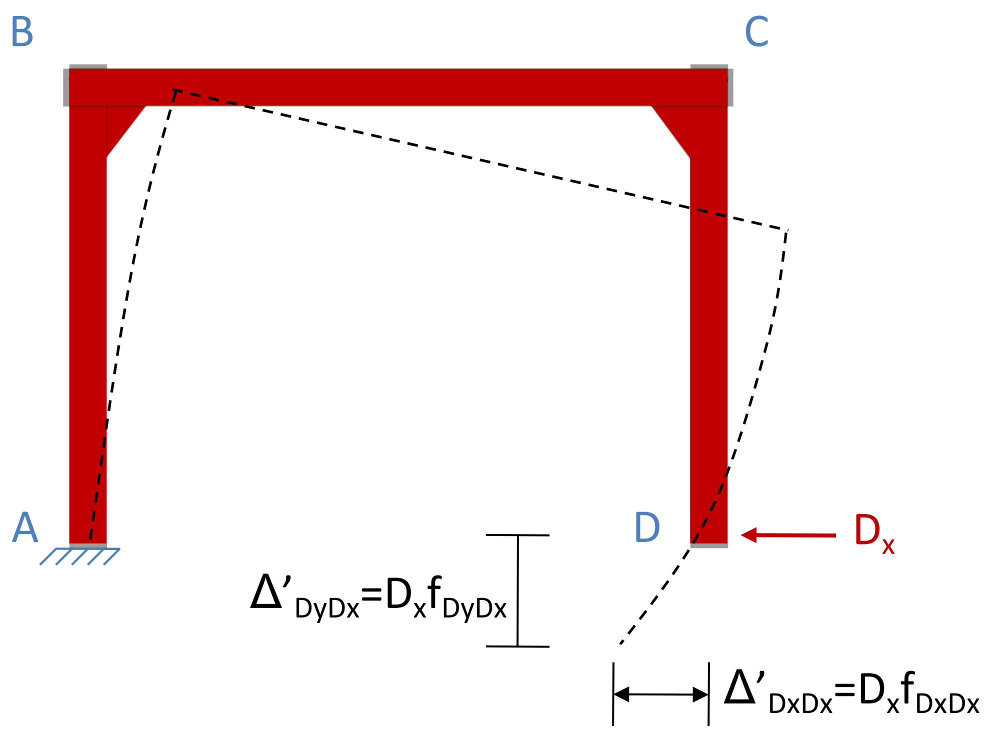


Primary Structure

Solution

Principle of Superposition

- Primary structure is subjected separately to the external loading and redundants D_x and D_y as shown.

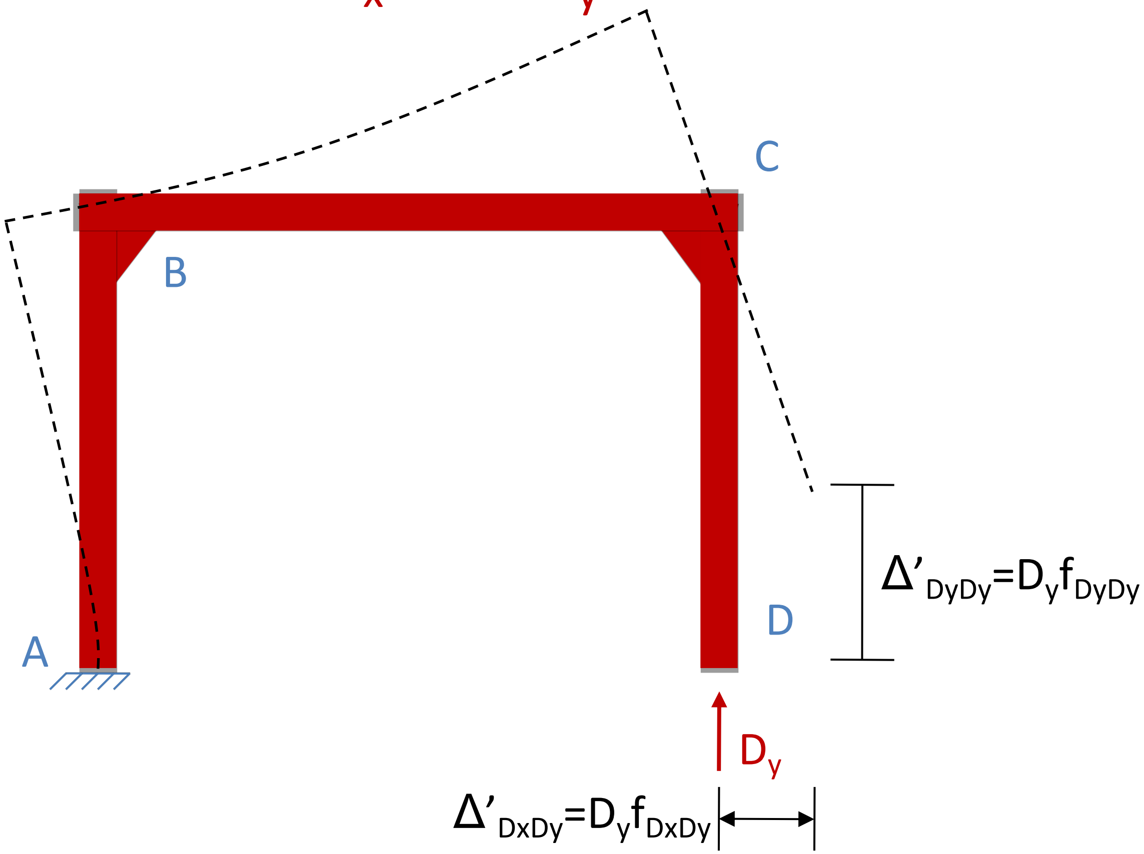


Redundant D_x applied

Solution

Principle of Superposition

- Primary structure is subjected separately to the external loading and redundants D_x and D_y as shown.



Redundant D_y applied

Solution

Compatibility Equation

$$0 = \Delta_{Dx} + \Delta'_{DxDx} + \Delta'_{DxDy} = \Delta_{Dx} + D_x f_{DxDx} + D_y f_{DxDy} \qquad (1)$$

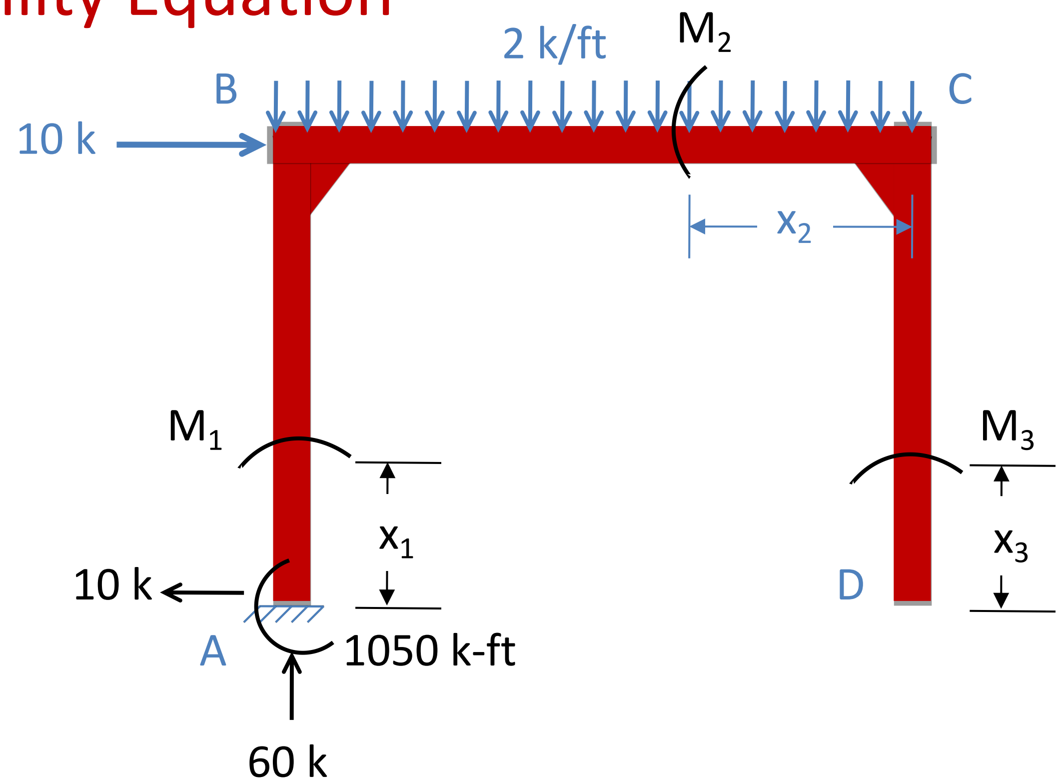
$$0 = \Delta_{Dy} + \Delta'_{DyDx} + \Delta'_{DyDy} = \Delta_{Dy} + D_x f_{DyDx} + D_y f_{DyDy} \qquad (2)$$

- The equations for bending moments for the members of the frame due to external loading and unit values of the redundants are tabulated in the table.

- By applying the virtual work method, we will find Δ_{Dx}, Δ_{Dy}, f_{DxDx}, f_{DyDx}, f_{DxDy}, f_{DyDy},

Solution

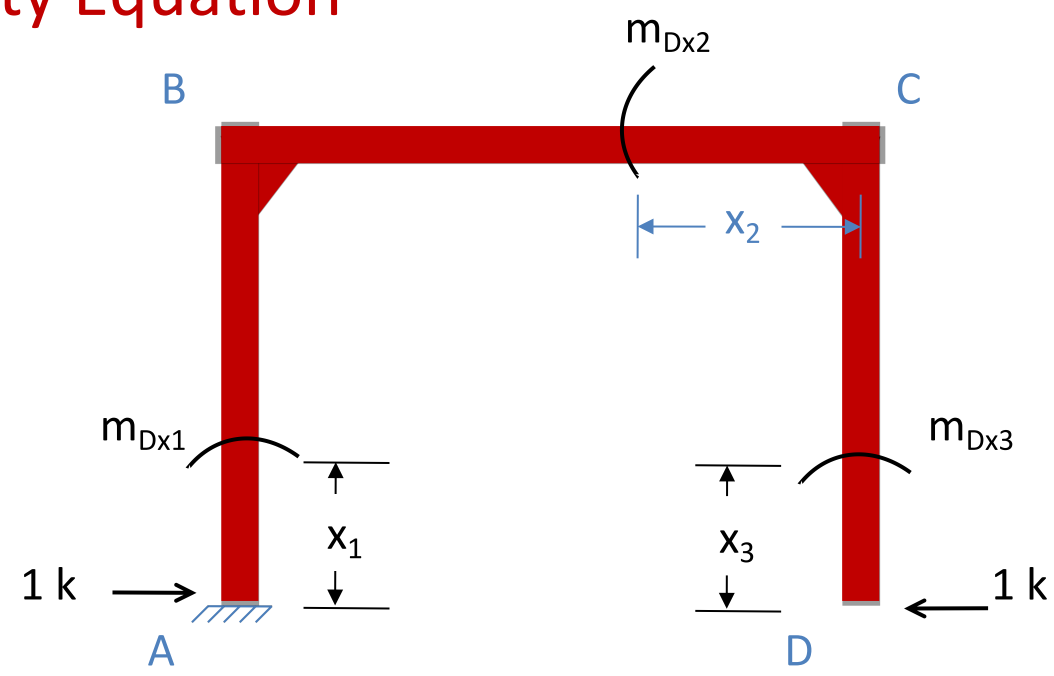
Compatibility Equation



Member	Origin	Limits	M (k-ft)		
AB	A	0-15	$-1050+10x_1$		
CB	C	0-30	$-x_2^2$		
DC	D	0-15	0		

Solution

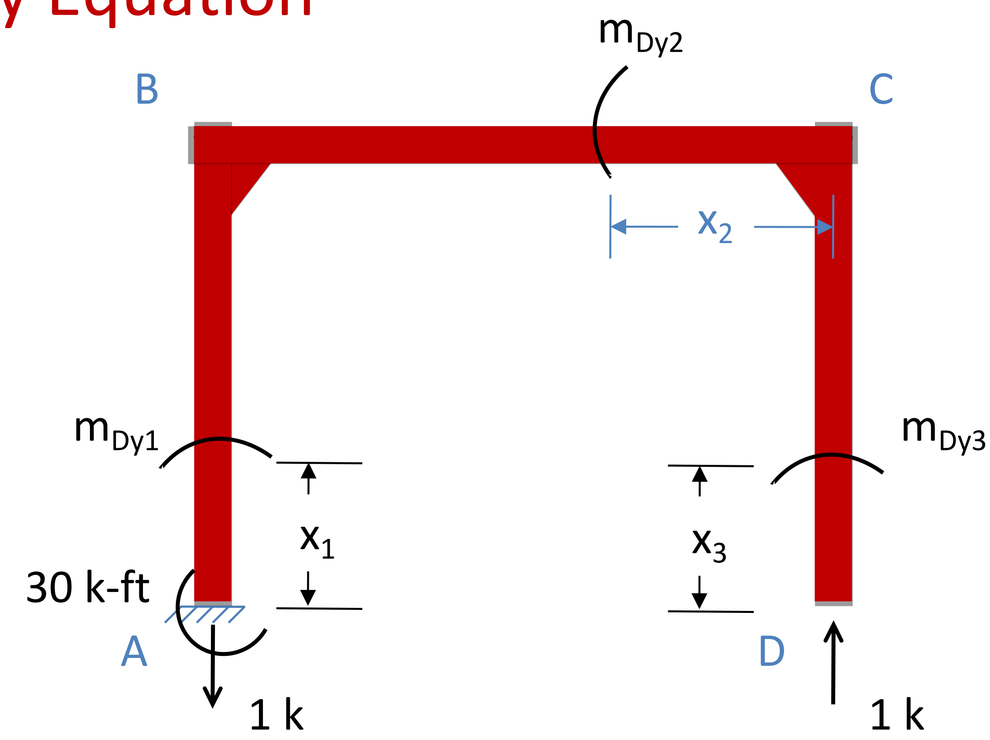
Compatibility Equation



Member	Origin	Limits	M (k-ft)	m_{Dx} (k-ft/k)	
AB	A	0-15	$-1050+10x_1$	$-x_1$	
CB	C	0-30	$-x_2^2$	-15	
DC	D	0-15	0	$-x_3$	

Solution

Compatibility Equation



Member	Origin	Limits	M (k-ft)	m_{Dx} (k-ft/k)	m_{Dy} (k-ft/k)
AB	A	0-15	$-1050+10x_1$	$-x_1$	30
CB	C	0-30	$-x_2^2$	-15	x_2
DC	D	0-15	0	$-x_3$	0

Solution

$$\Delta_{Dx} = \int_0^L \frac{Mm_{Dx}}{EI}dx = \int_0^{15} \frac{(-1050+10x_1)(-x_1)}{EI}dx_1 + \int_0^{30} \frac{(-x_2^2)(-15)}{EI}dx_2 + \int_0^{15} \frac{(0)(-x_3)}{EI}dx_3$$

$$\Delta_{Dx} = 106875 + 135000 + 0 = \frac{241875}{EI} k - ft^3$$

$$\Delta_{Dy} = \int_0^L \frac{Mm_{Dy}}{EI}dx = \int_0^{15} \frac{(-1050+10x_1)(30)}{EI}dx_1 + \int_0^{30} \frac{(-x_2^2)(x_2)}{EI}dx_2 + 0$$

$$\Delta_{Dy} = -438750 - 202500 + 0 = -\frac{641250}{EI} k - ft^3$$

Solution

$$f_{DxDx} = \int_0^L \frac{m_{Dx} m_{Dx}}{EI} dx = \int_0^{15} \frac{(-x_1)^2}{EI} dx_1 + \int_0^{30} \frac{(-15)^2}{EI} dx_2 + \int_0^{15} \frac{(-x_3)^2}{EI} dx_3$$

$$f_{DxDx} = \frac{9000}{EI} ft^3$$

$$f_{DyDy} = \int_0^L \frac{m_{Dy} m_{Dy}}{EI} dx = \int_0^{15} \frac{(30)^2}{EI} dx_1 + \int_0^{30} \frac{(x_2)^2}{EI} dx_2$$

$$f_{DyDy} = \frac{22500}{EI} ft^3$$

Solution

$$f_{DxDy} = f_{DyDx} = \int_0^L \frac{m_{Dx} m_{Dy}}{EI} dx = \int_0^{15} \frac{(-x_1)(30)}{EI} dx_1 + \int_0^{30} \frac{(-15)(x_2)}{EI} dx_2$$

$$f_{DxDy} = f_{DyDx} = -\frac{10125}{EI} ft^3$$

Solution

$$\Delta_{Dx} = \frac{241875}{EI} k - ft^3$$

$$\Delta_{Dy} = -\frac{641250}{EI} k - ft^3$$

$$f_{DxDx} = \frac{9000}{EI} ft^3$$

$$f_{DyDy} = \frac{22500}{EI} ft^3$$

$$f_{DxDy} = f_{DyDx} = -\frac{10125}{EI} ft^3$$

Solution

Now put these values in the Equations (1) and (2)

$$0 = 241875 + 9000D_x - 10125D_y \qquad (1)$$

$$0 = -641250 - 10125D_x + 22500D_y \qquad (2)$$

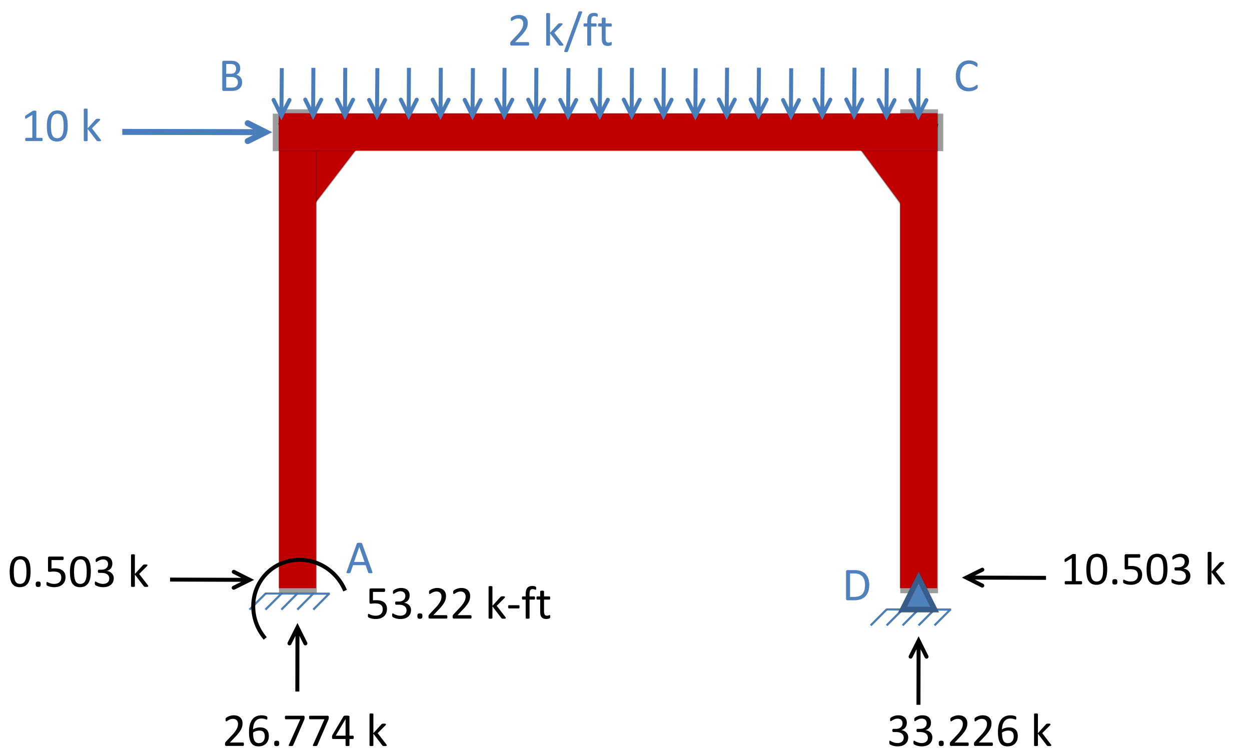
By solving (1) and (2) simultaneously we get

$$D_x = 10.503\,k \leftarrow$$

$$D_y = 33.226\,k \uparrow$$

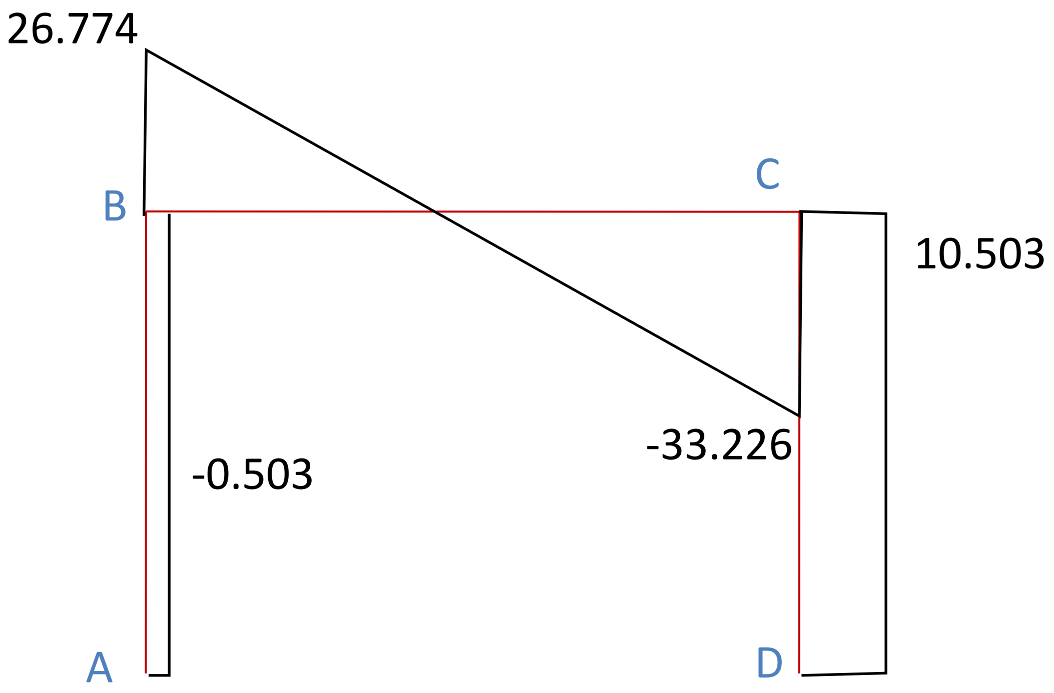
Solution

- Applying equations of equilibrium, we have the other support reactions as



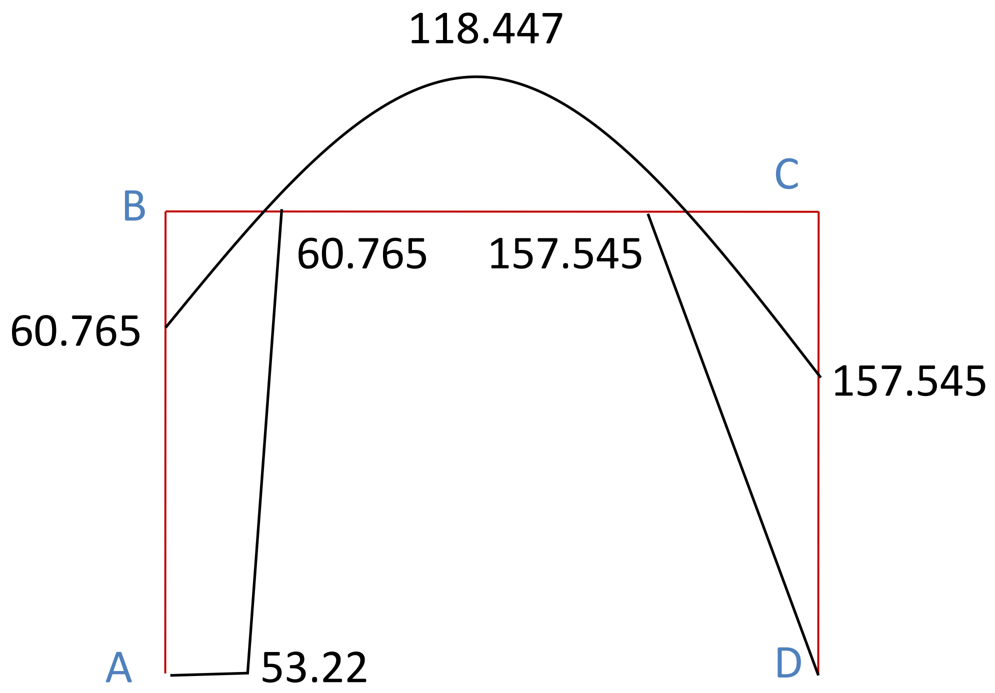
Solution

- Shear diagram



Solution

- Moment diagram



TRUSSES

- The degree of indeterminacy of a truss can be find using Equation $b+r > 2j$.

  where

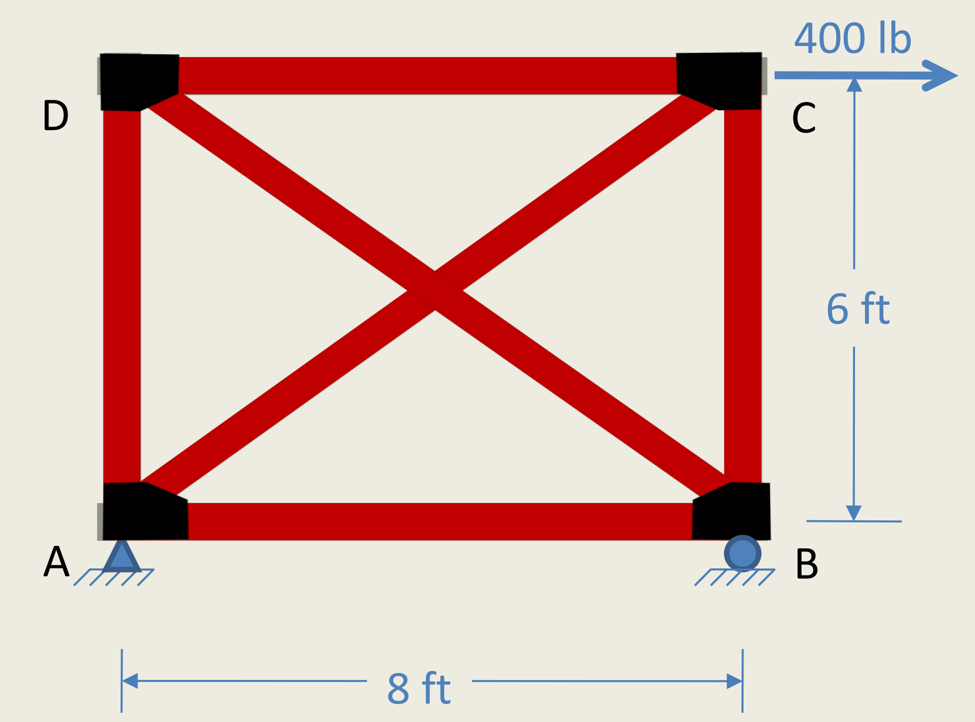
  b = unknown bar forces, r = support reactions,

  2j = equations of equilibrium

- This method is quite suitable for analyzing trusses that are statically indeterminate to the first or second degree.

Example 9

Determine the force in member AC of the truss shown. AE is same for all members.



Actual Truss

Solution

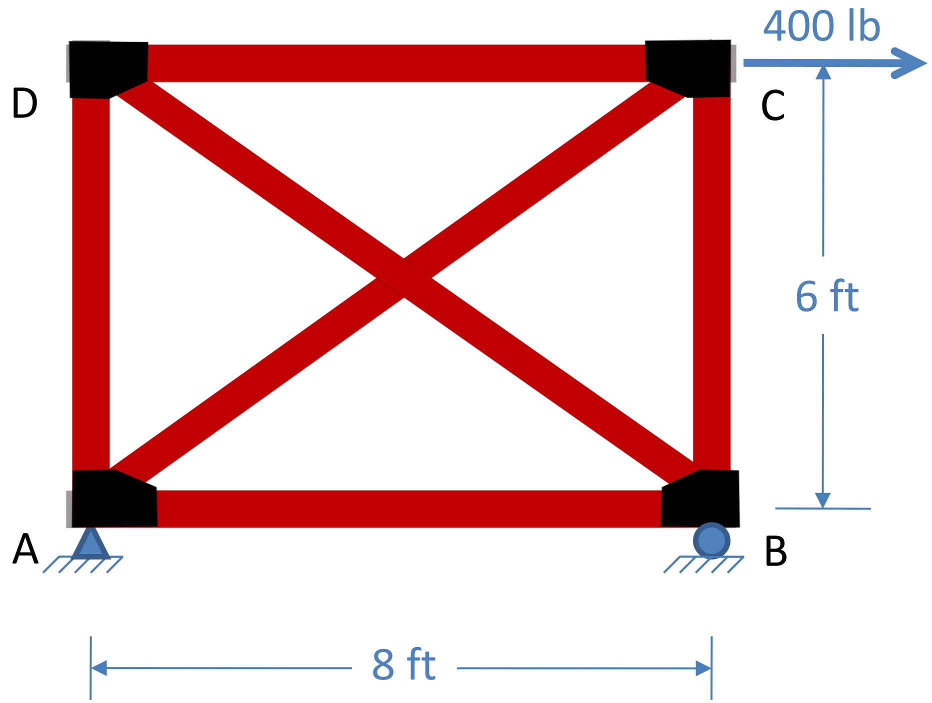
The truss is statically indeterminate to the first degree.

$b + r = 2j$

$6 + 3 = 2(4)$

$9 > 8$

$9 - 8 = 1^{st}$ degree

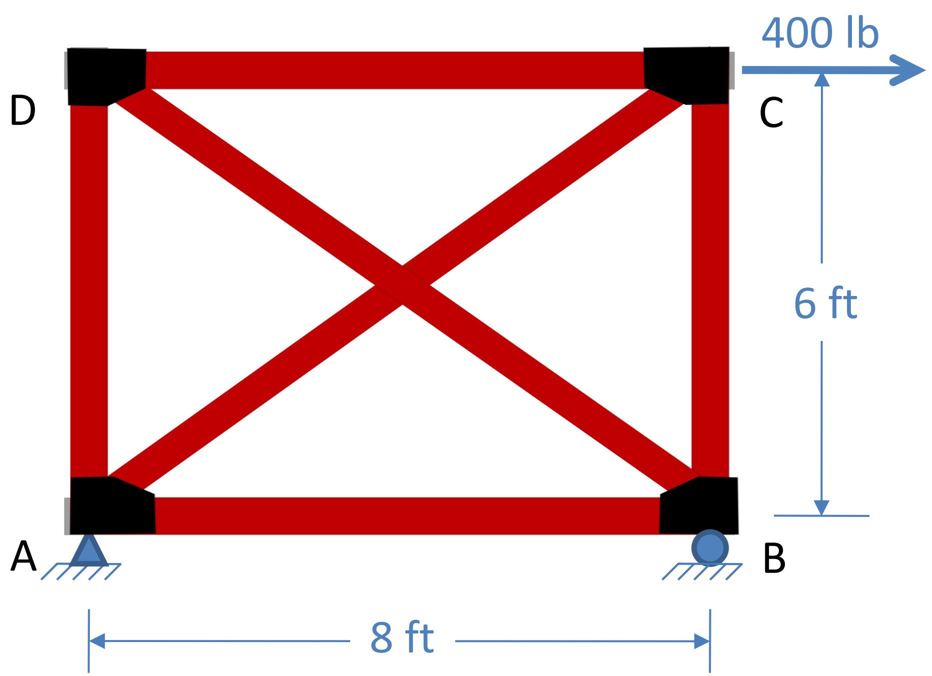


Actual Truss

Solution

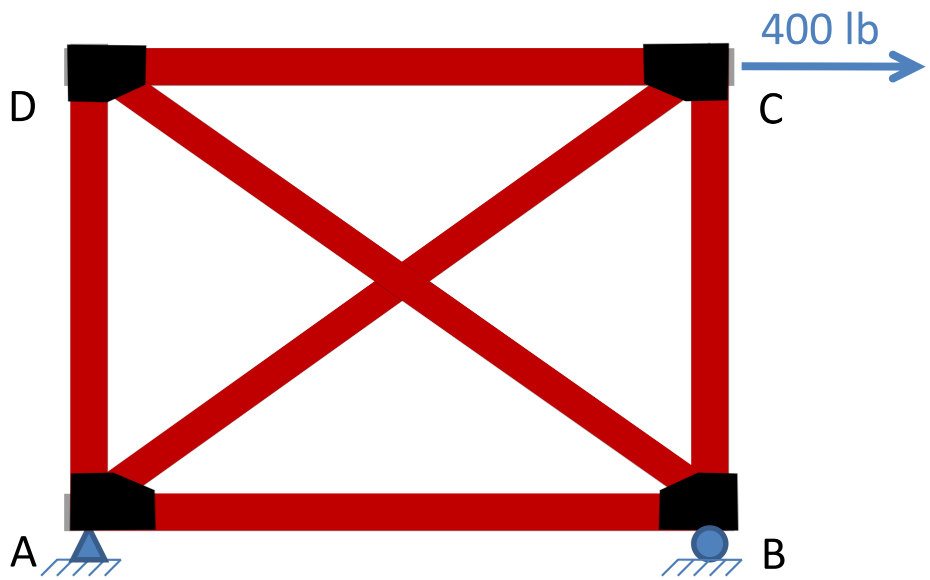
Principle of Superposition

- The force in member AC is to be determined, so member AC is chosen as redundant.
- This requires cutting this member, so that it cannot sustain a force, making the truss S.D. and stable.



Actual Truss

Solution



Actual Truss

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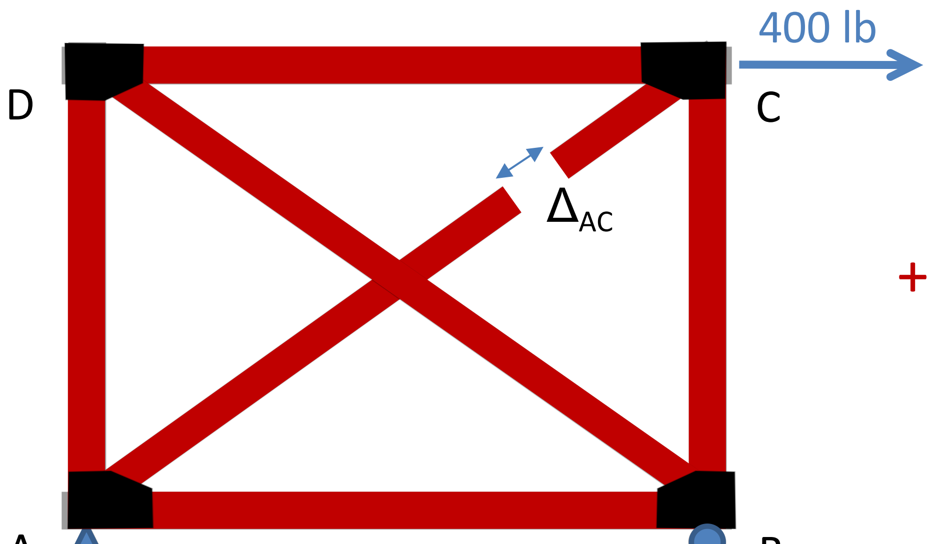
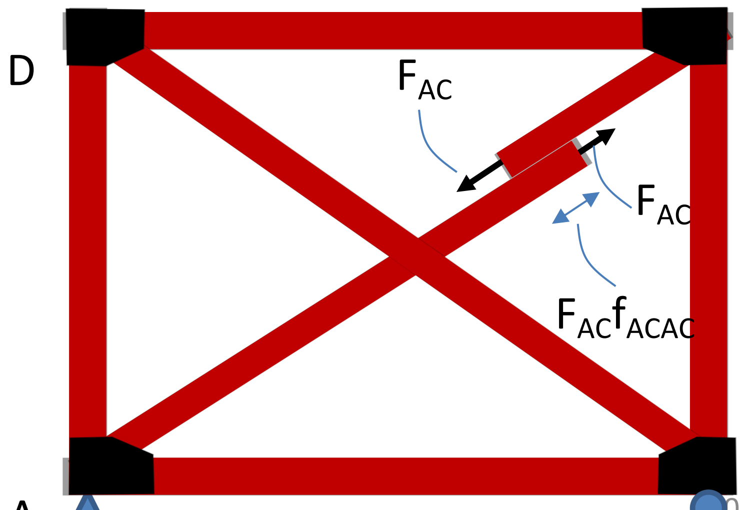


Primary Structure

+



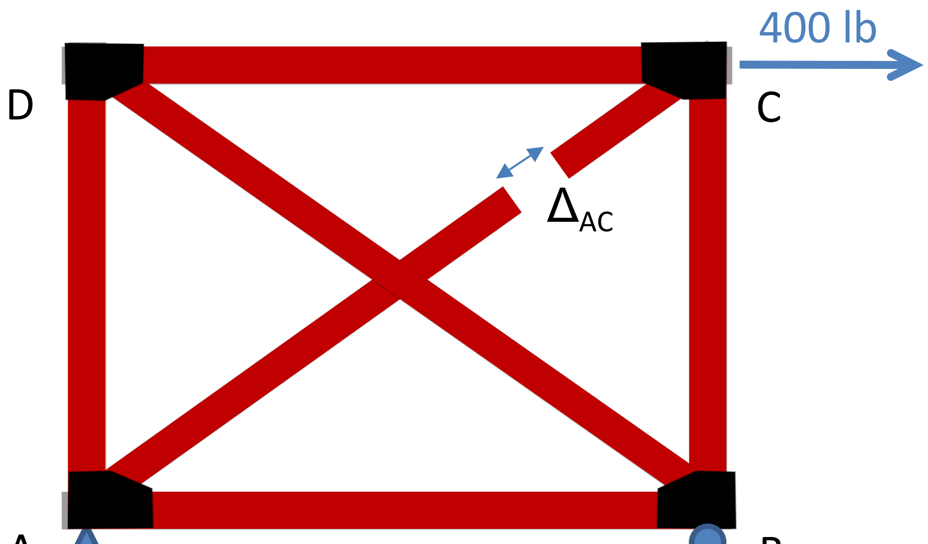
Redundant F_{AC} applied

Solution

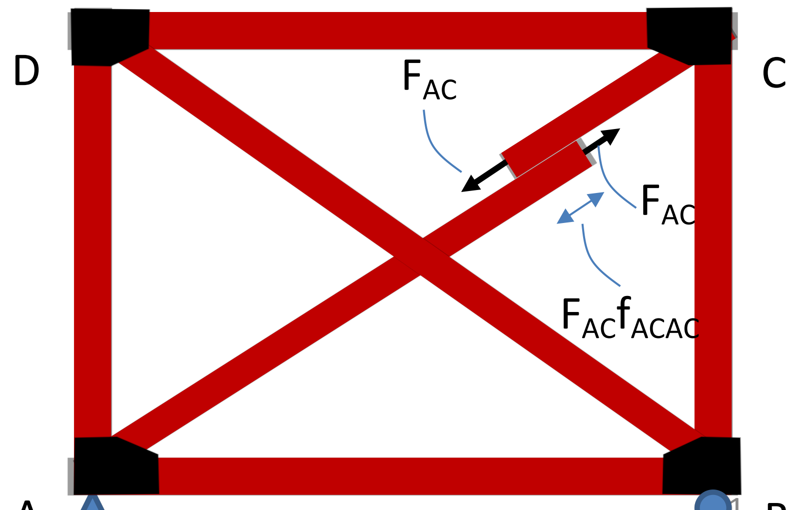
Compatibility Equation

- With reference to member AC, we require the relative displacement Δ_{AC}, which occurs at the ends of cut member AC due to the 400-lb load, plus the relative displacement $F_{AC}f_{ACAC}$ caused by the redundant force acting alone, be equal to zero, that is

$$0 = \Delta_{AC} + F_{AC} f_{ACAC}$$



Primary Structure



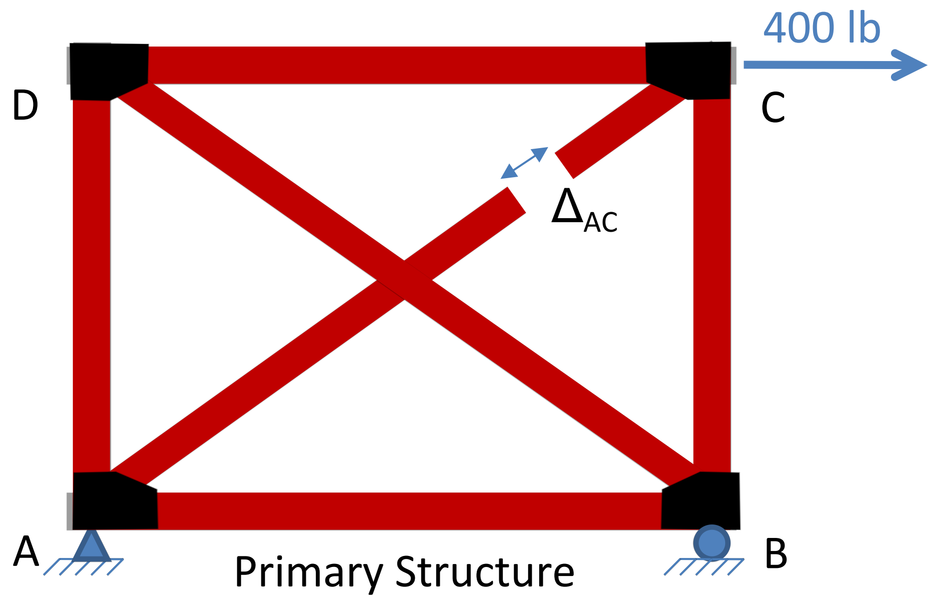
Redundant F_{AC} applied

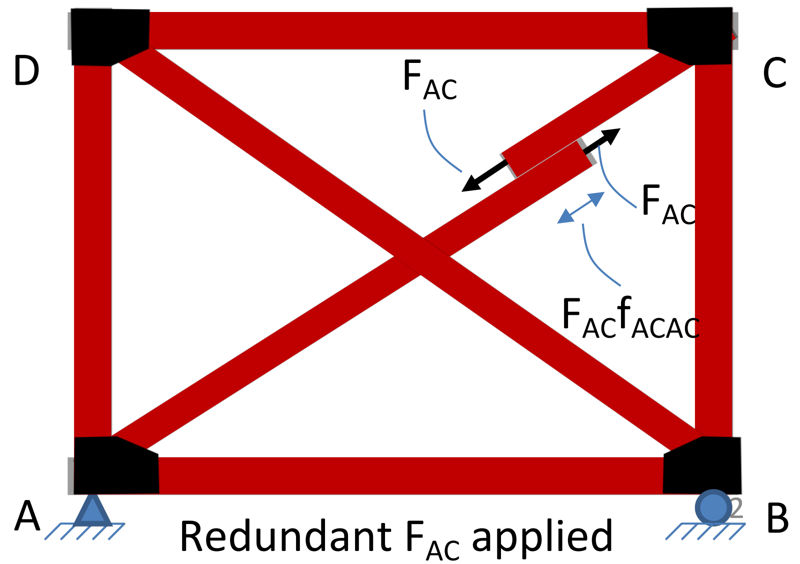
Solution

Compatibility Equation

- Here the flexibility coefficient f_{ACAC} represents the relative displacement of the cut ends of member AC caused by a real unit load acting at the cut ends of member AC.

$$0 = \Delta_{AC} + F_{AC} f_{ACAC}$$



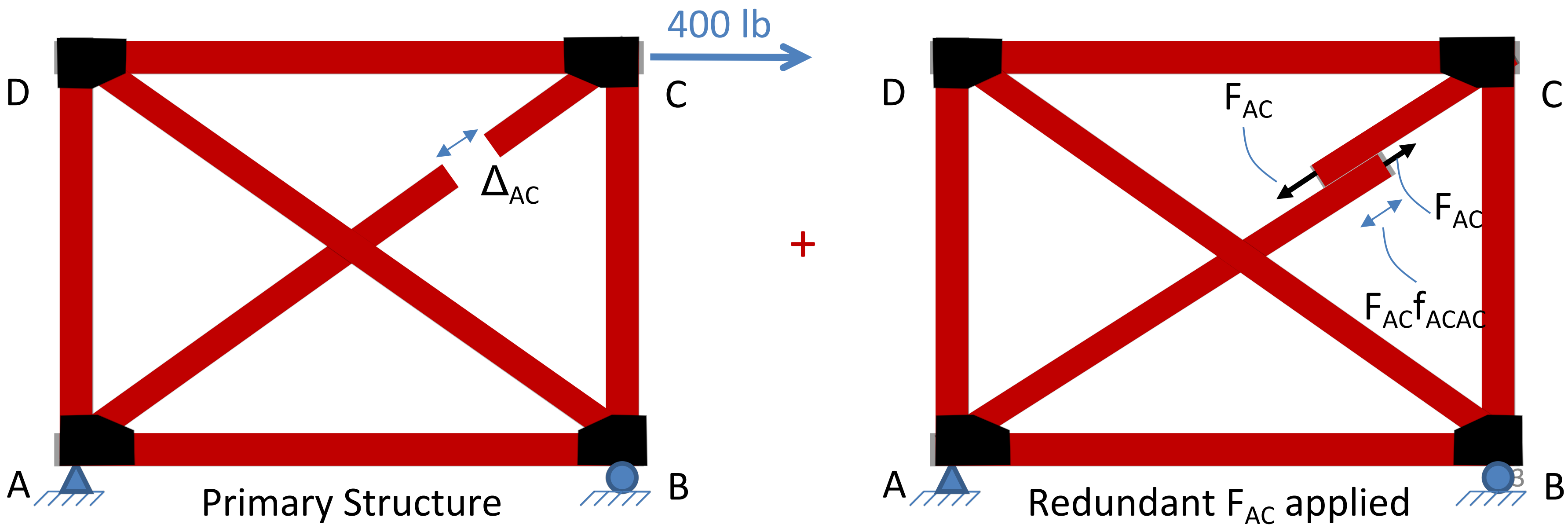


Solution

Compatibility Equation

- This term, f_{ACAC}, and Δ_{AC} will be computed using the method of virtual work.

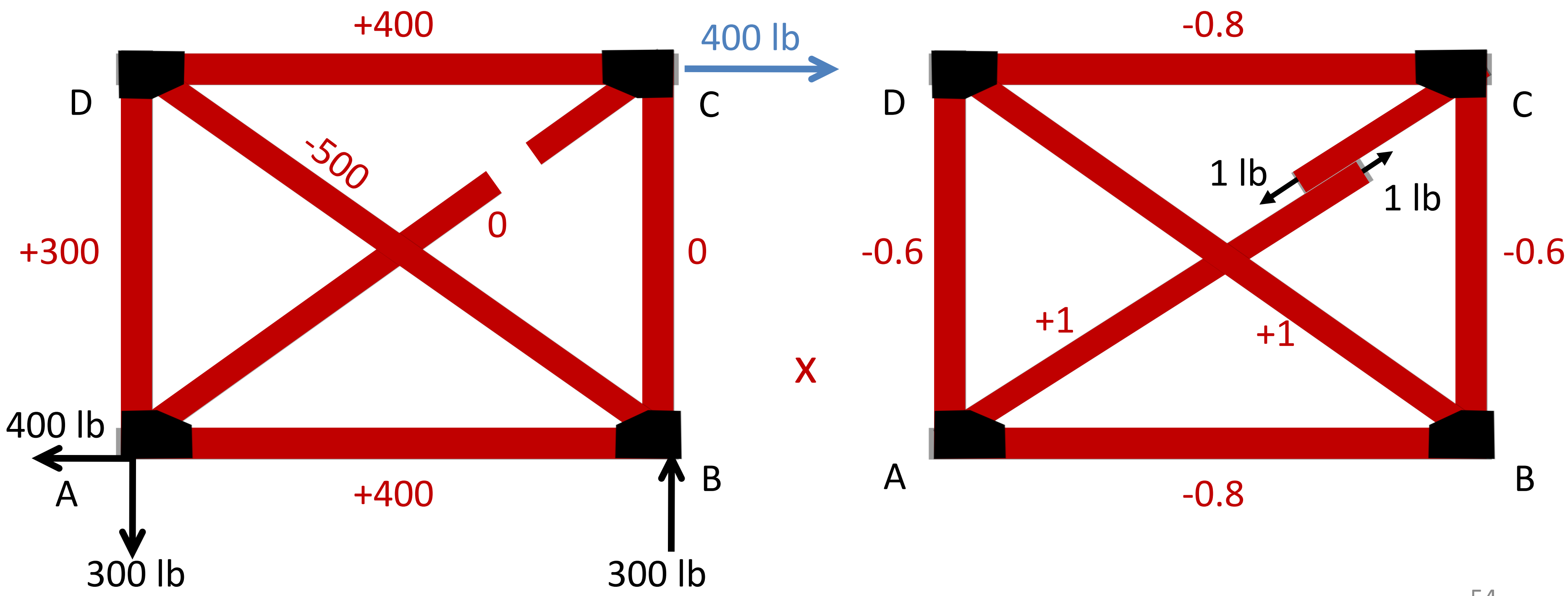
$$0 = \Delta_{AC} + F_{AC} f_{ACAC}$$



Solution

Compatibility Equation

- For Δ_{AC} we require application of the real load of 400 lb, and a virtual unit force acting at the cut ends of member AC.

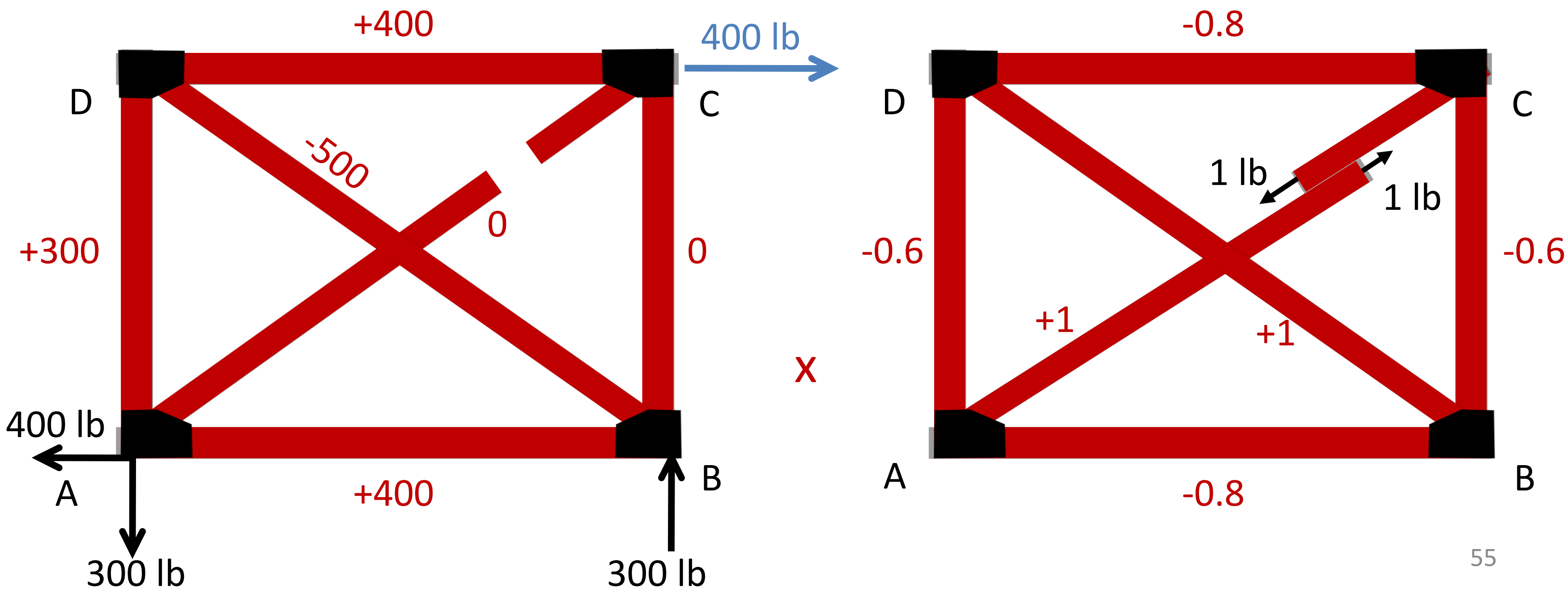


Solution

$$\Delta_{AC} = \sum \frac{nNL}{AE}$$

$$= 2\left[\frac{(-0.8)(400)(8)}{AE}\right] + \frac{(-0.6)(0)(6)}{AE} + \frac{(-0.6)(300)(6)}{AE} + \frac{(1)(-500)(10)}{AE} + \frac{(1)(0)(10)}{AE}$$

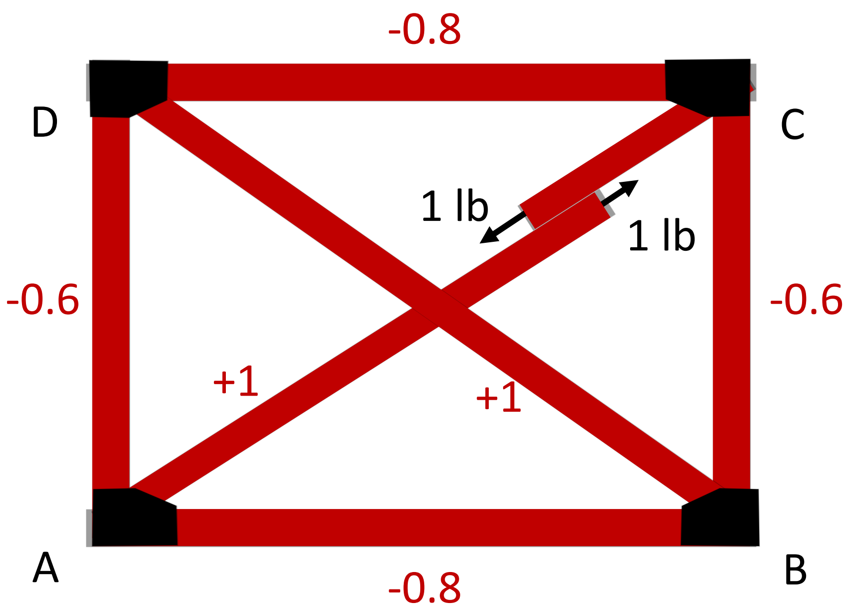
$$= -\frac{11200}{AE}$$



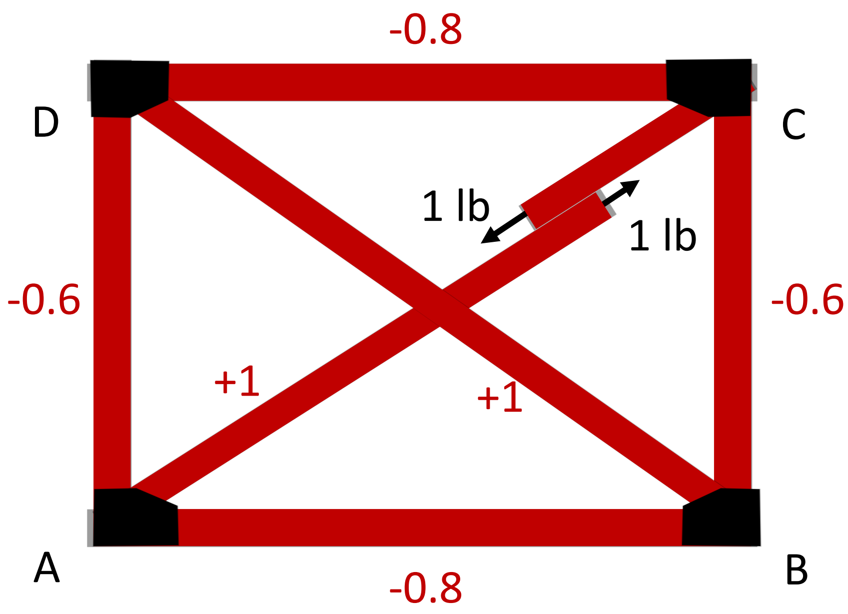
Solution

Compatibility Equation

- For f_{ACAC} we require application of the real unit forces acting on the cut ends of member AC, and virtual unit forces acting on the cut ends of member AC



x

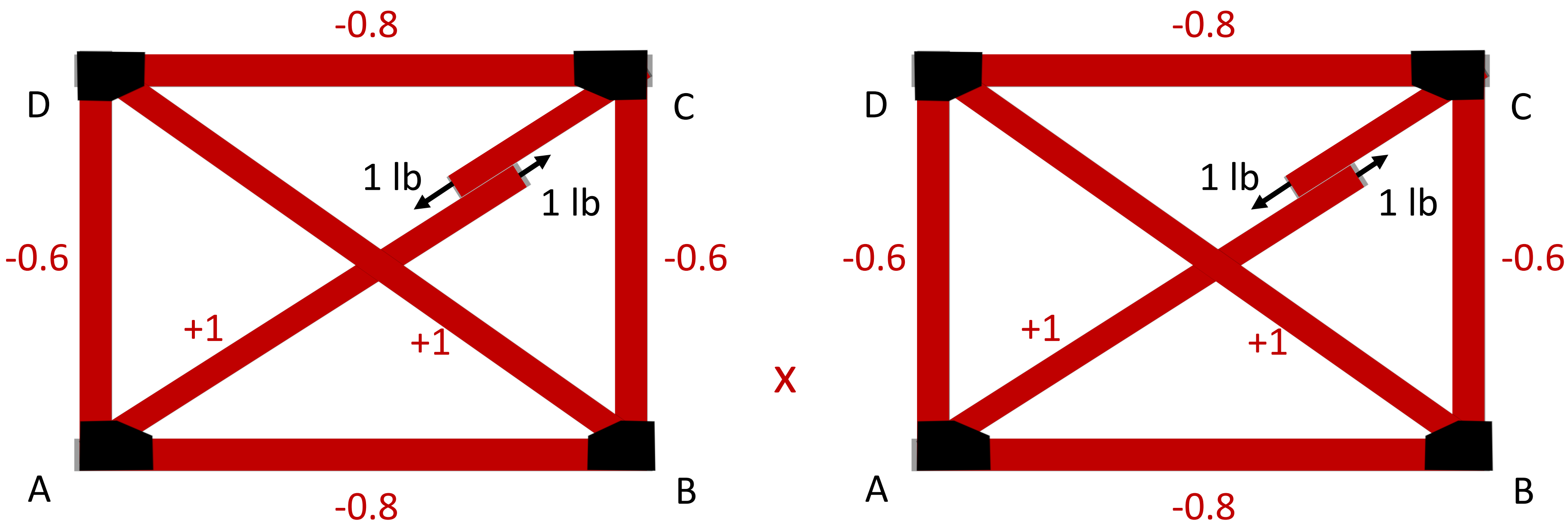


Solution

$$f_{ACAC} = \sum \frac{n^2 L}{AE}$$

$$= 2\left[\frac{(-0.8)^2(8)}{AE}\right] + 2\left[\frac{(-0.6)^2(6)}{AE}\right] + 2\left[\frac{(1)^2 10}{AE}\right]$$

$$= \frac{34.56}{AE}$$



Solution

Substituting the data into Eq. (1) and solving yields

$$0 = -\frac{11200}{AE} + \frac{34.56}{AE} F_{AC}$$

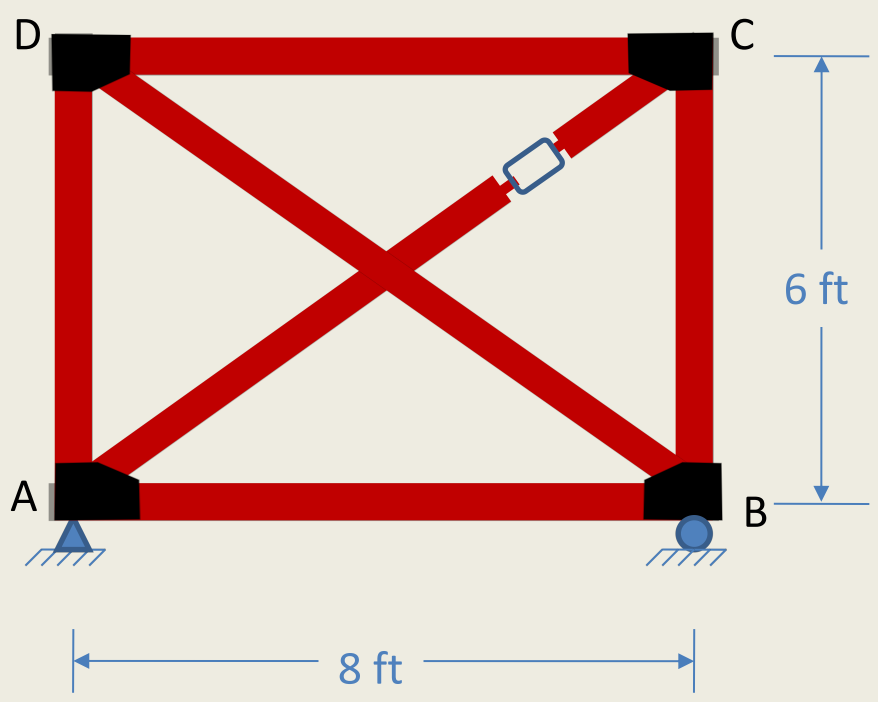
$$F_{AC} = 324\ \text{lb (T)} \qquad \text{ANS}$$

Since the numerical result is positive, AC is subjected to tension as assumed.

Using this result, the forces in other members can be found by equilibrium, using the method of joint.

Example 10

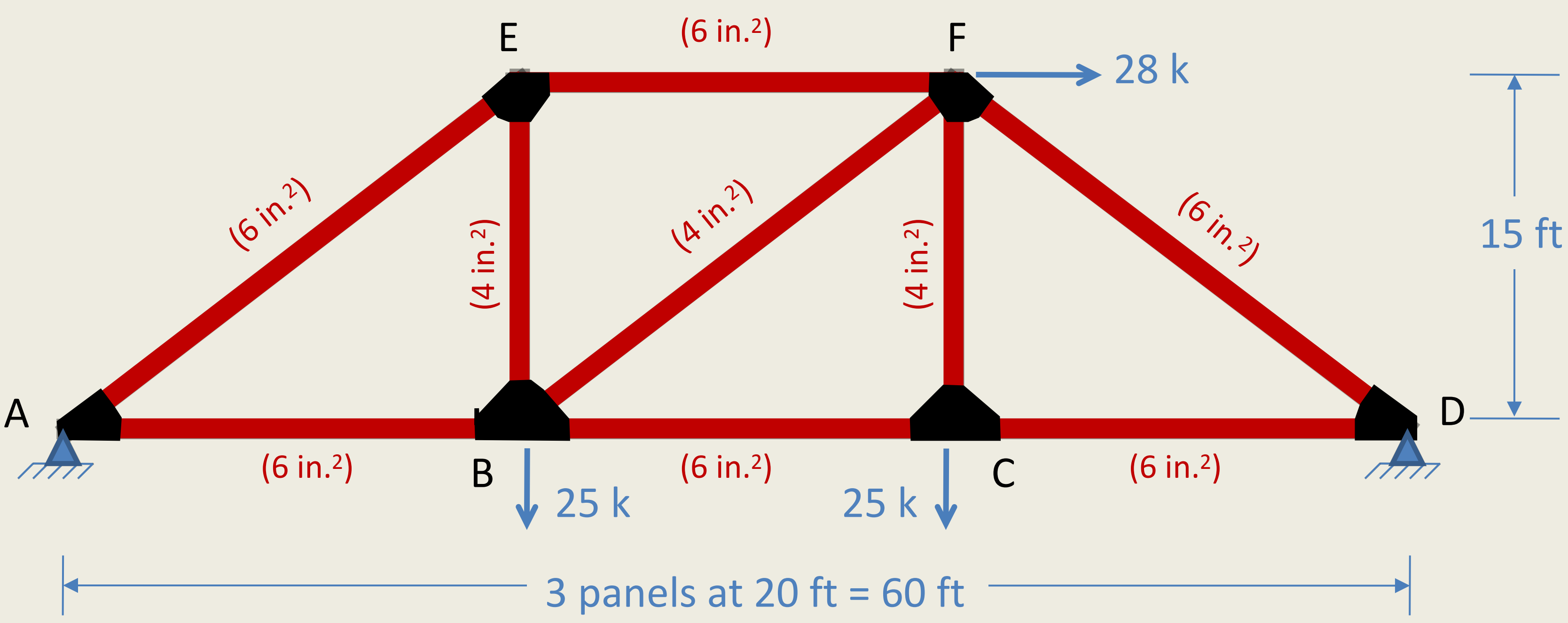
Determine the force in member AC of the truss shown.



Actual Truss

Example 11

Determine the reactions and the force in each member of the truss shown in Fig. shown. E = 29,000 ksi



Actual Truss

Solution

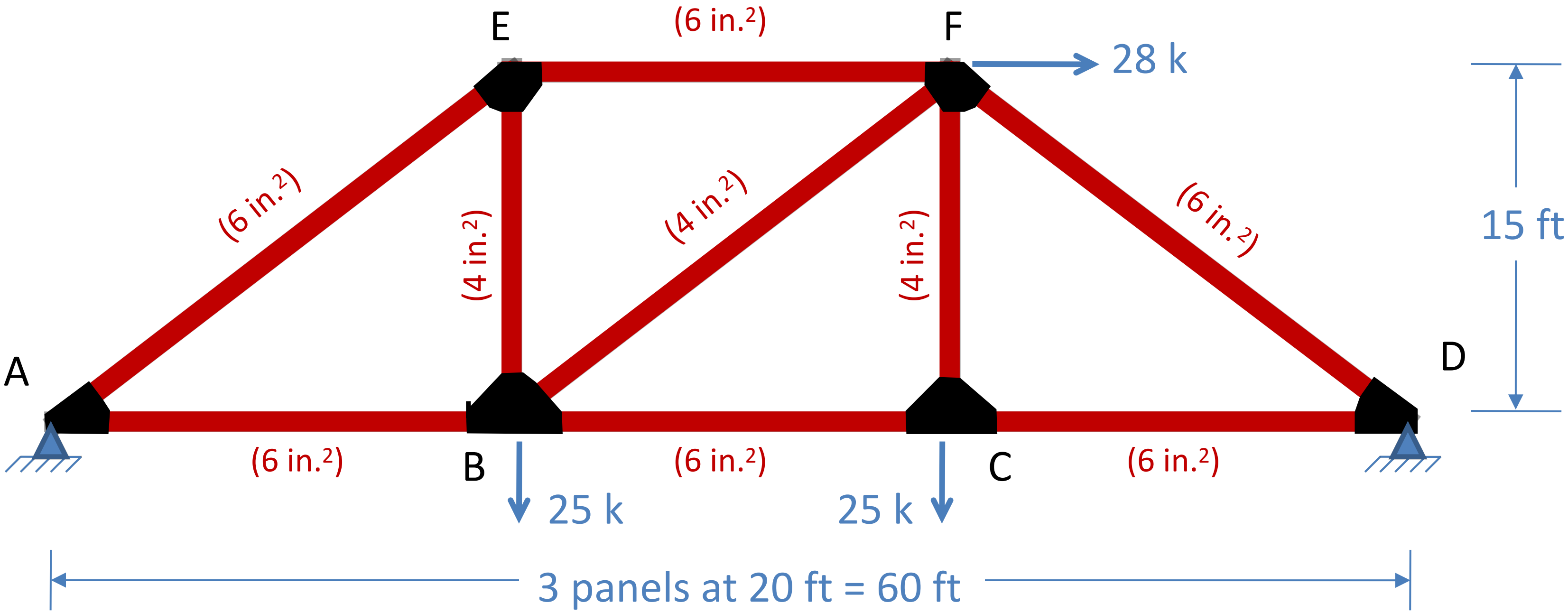
The truss is statically indeterminate to the first degree.

$b + r = 2j$

$9 + 4 = 2(6)$

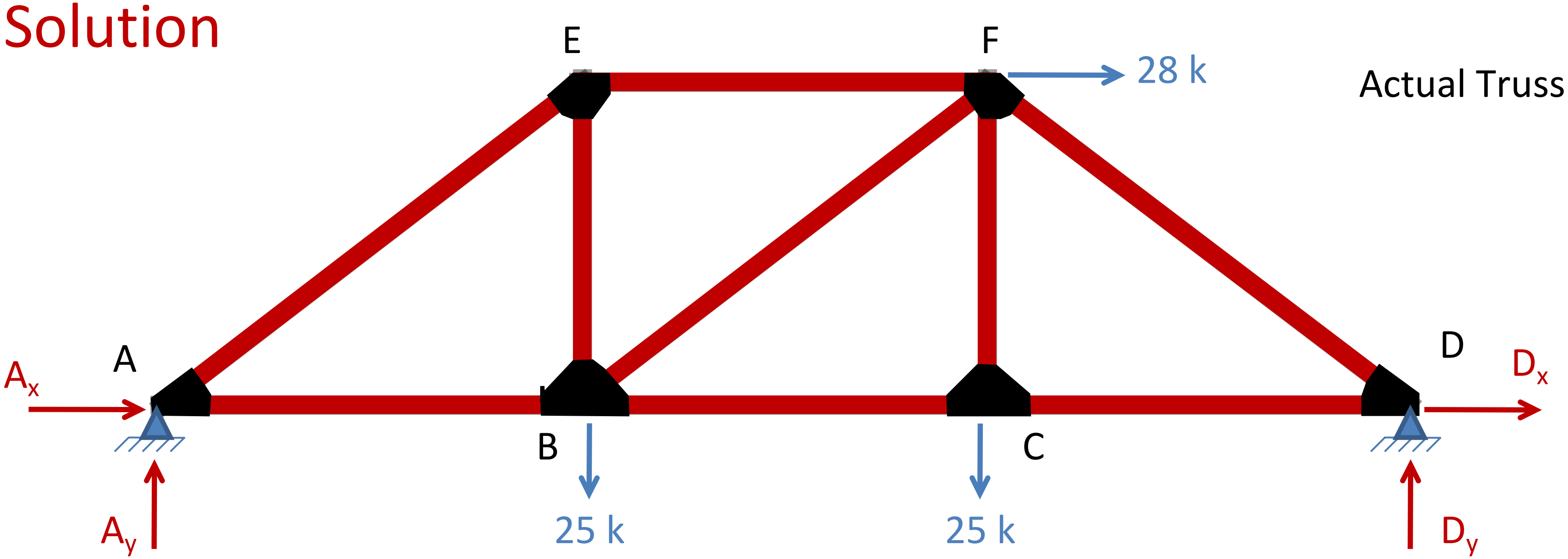
$13 > 12$

$13 - 12 = 1^{st}$ degree



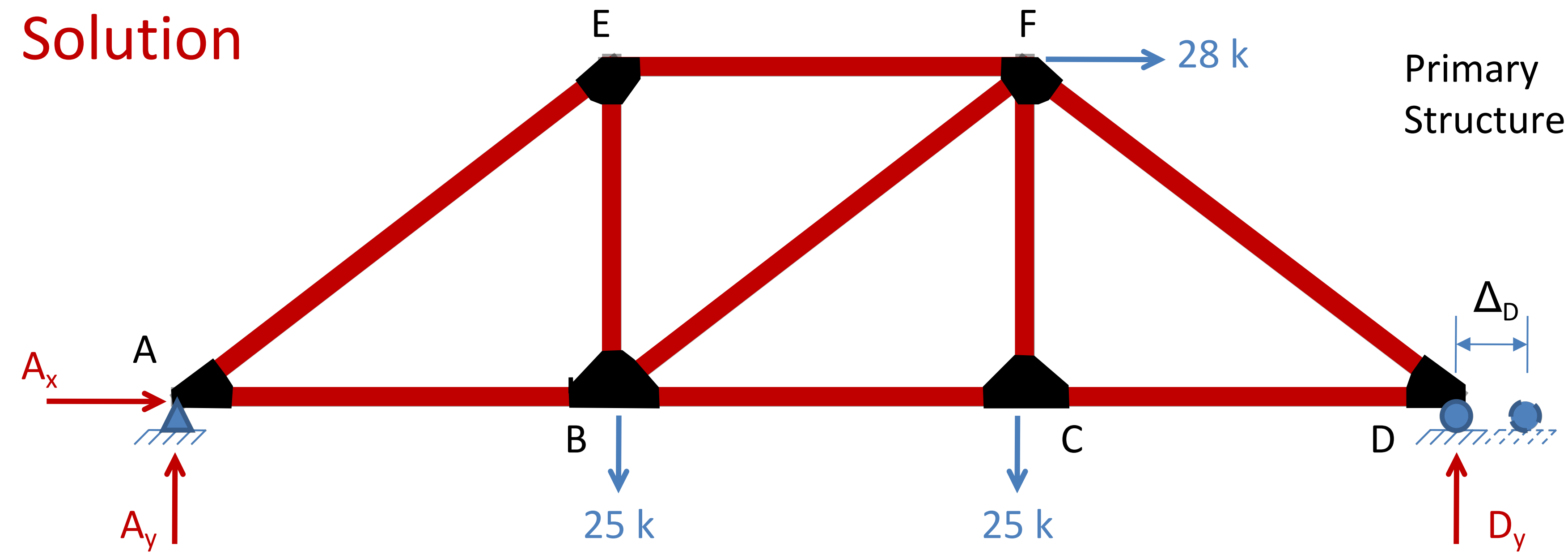
Actual Truss

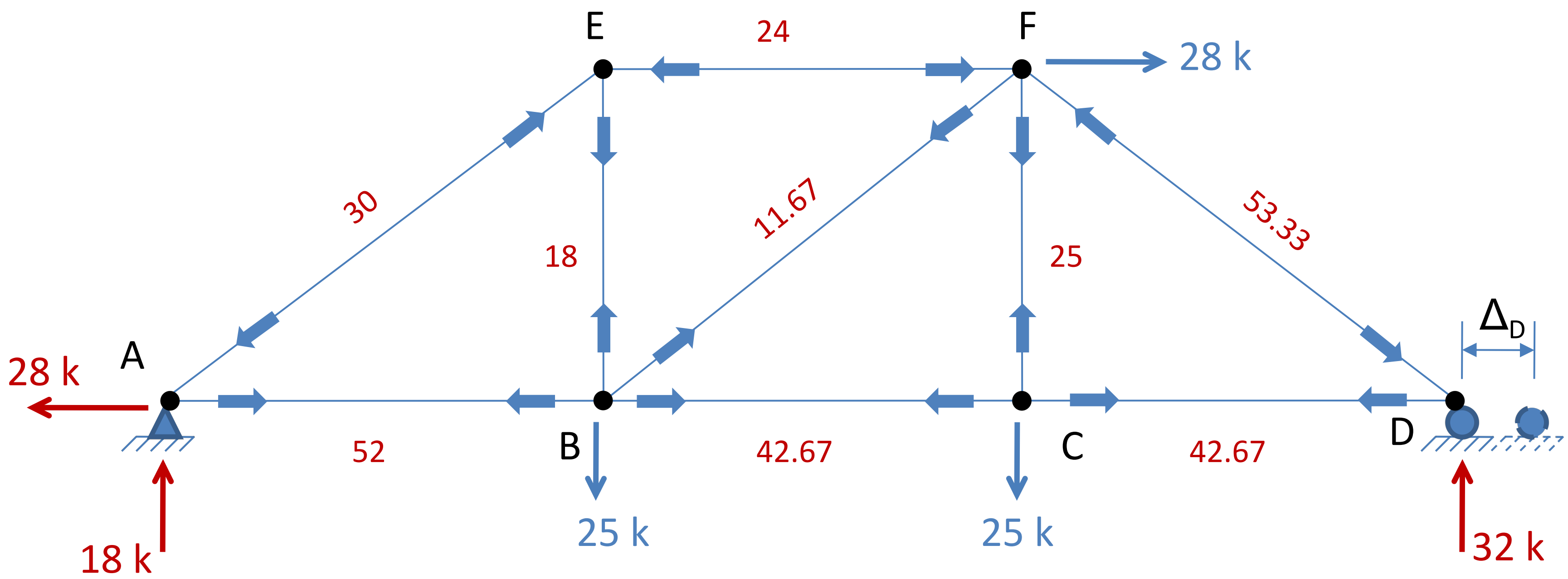
Solution



- D_x at hinged support D is selected as Redundant.
- Primary structure is obtained by removing the effect of D_x and replacing hinge by roller support there.
- Primary structure is subjected separately to external loading and redundant Force D_x.

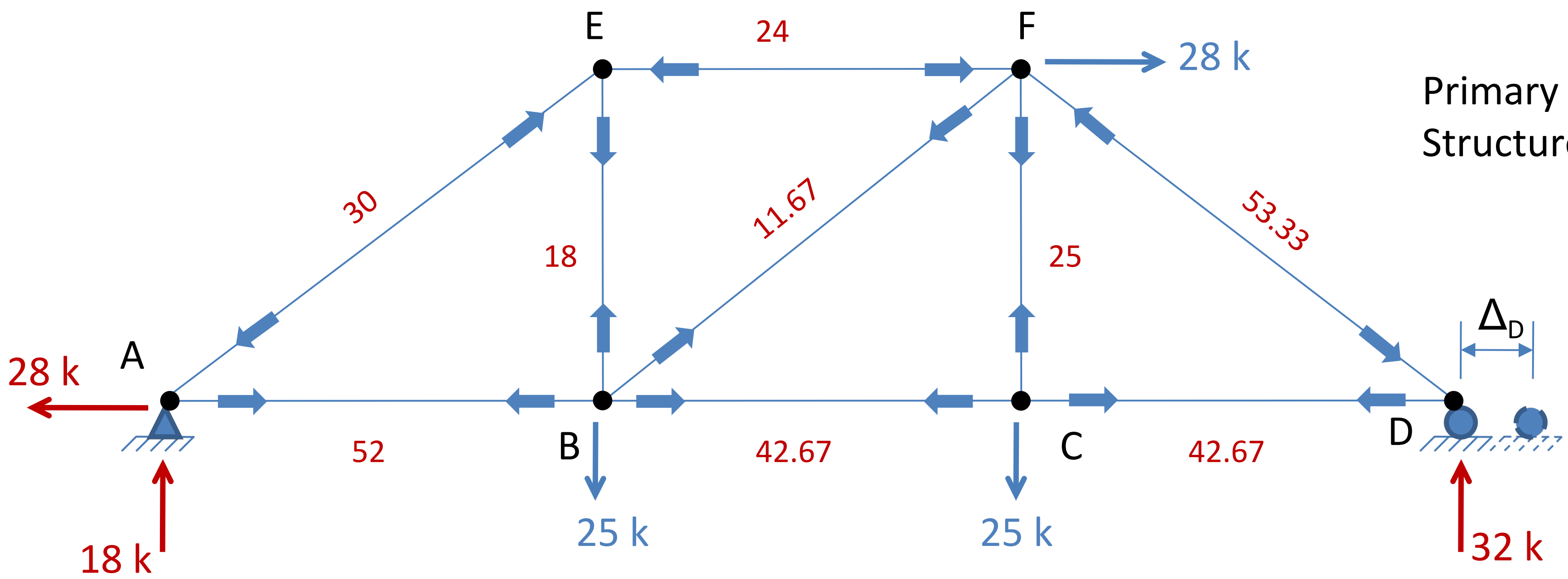
Solution


E
F
28 k
Primary Structure
A
A_x
B
C
D
Δ_D
A_y
25 k
25 k
D_y

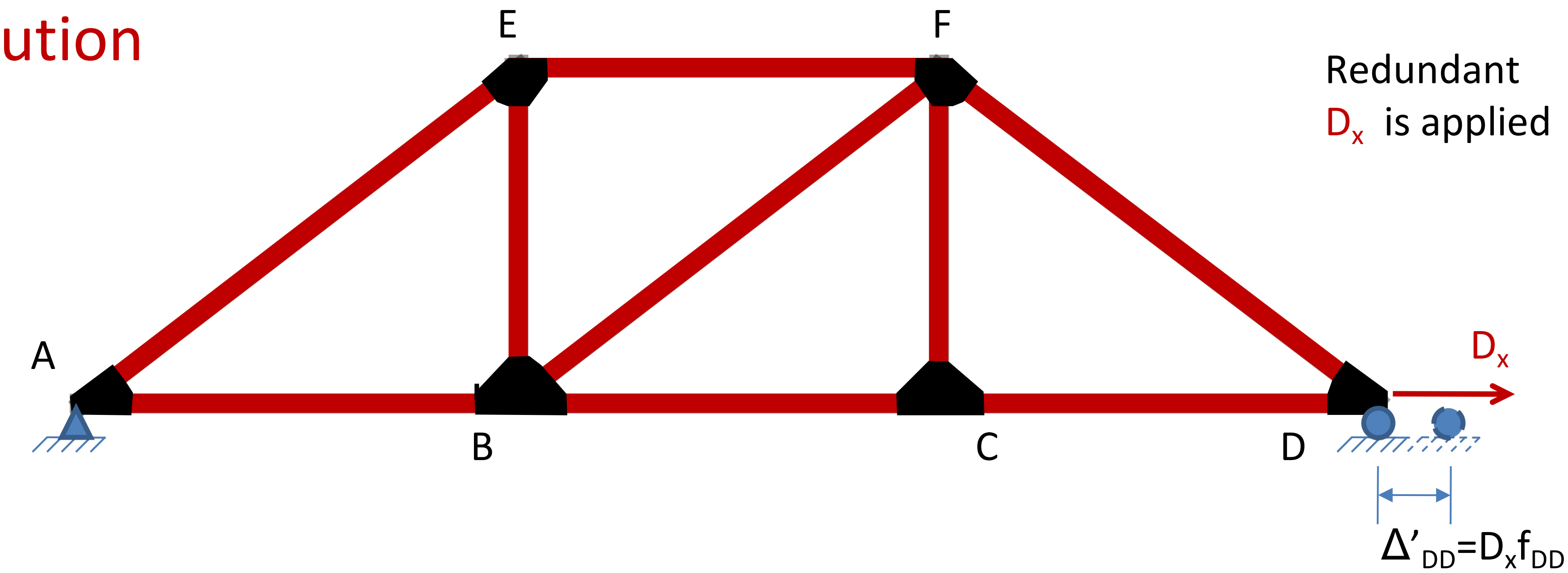

E
24
F
28 k
30
18
11.67
25
53.33
A
28 k
52
B
42.67
C
42.67
D
Δ_D
25 k
25 k
18 k
32 k

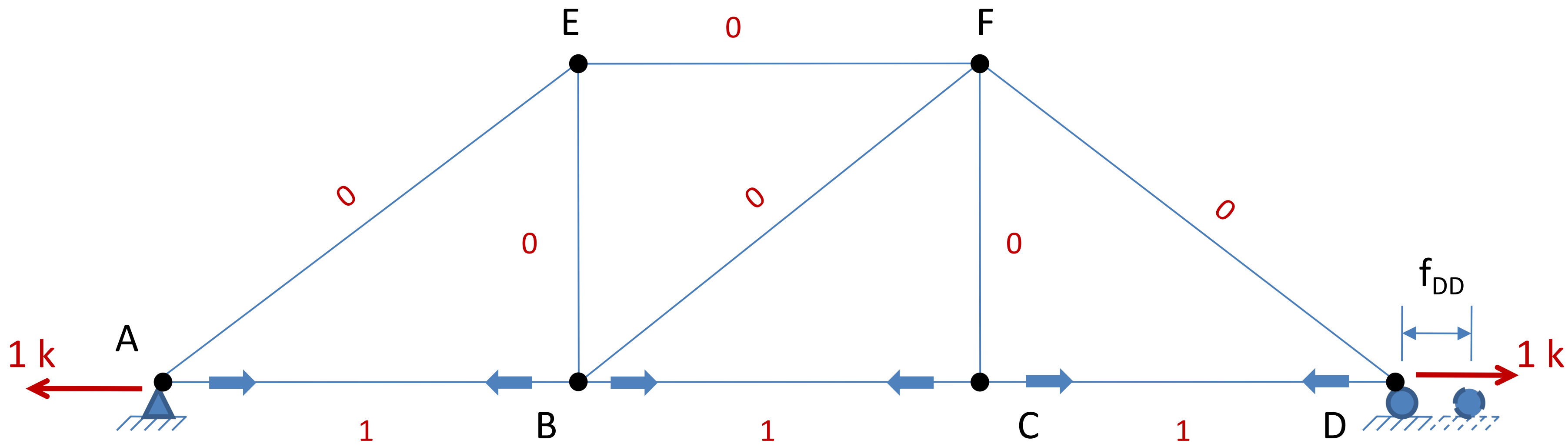
Solution

- Δ_D is horizontal deflection at point ' D ' of primary structure due to external loading.

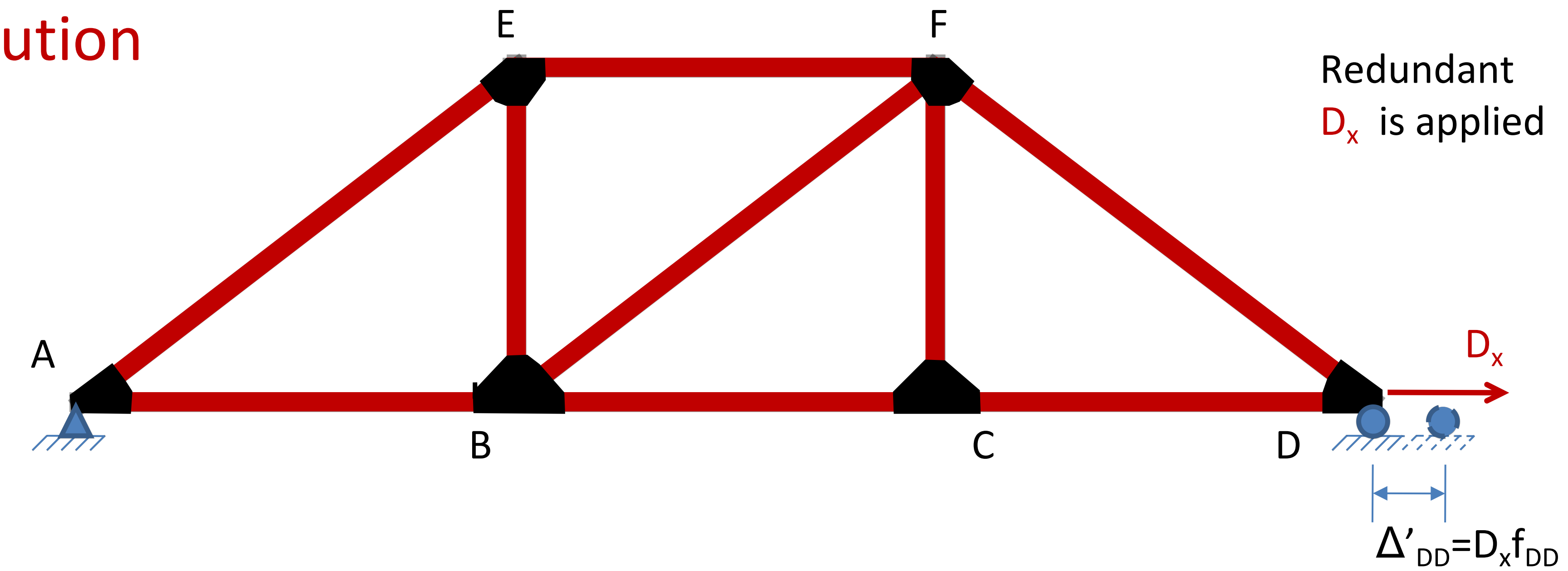


Solution


E
F
Redundant
Dx is applied
A
Dx
B
C
D
Δ'DD=DxfDD


E
0
F
0
0
0
0
0
fDD
1 k
A
1 k
1
B
1
C
1
D

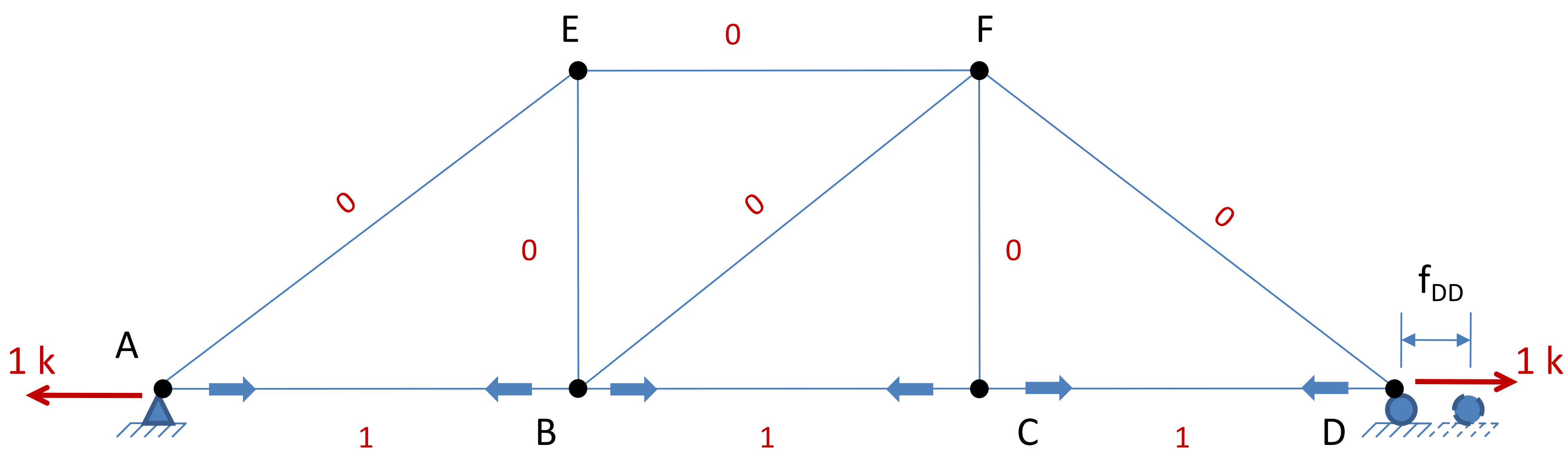
Solution



Δ'_{DD} is horizontal deflection at point ‘D’ due to redundant force D_x.

Solution

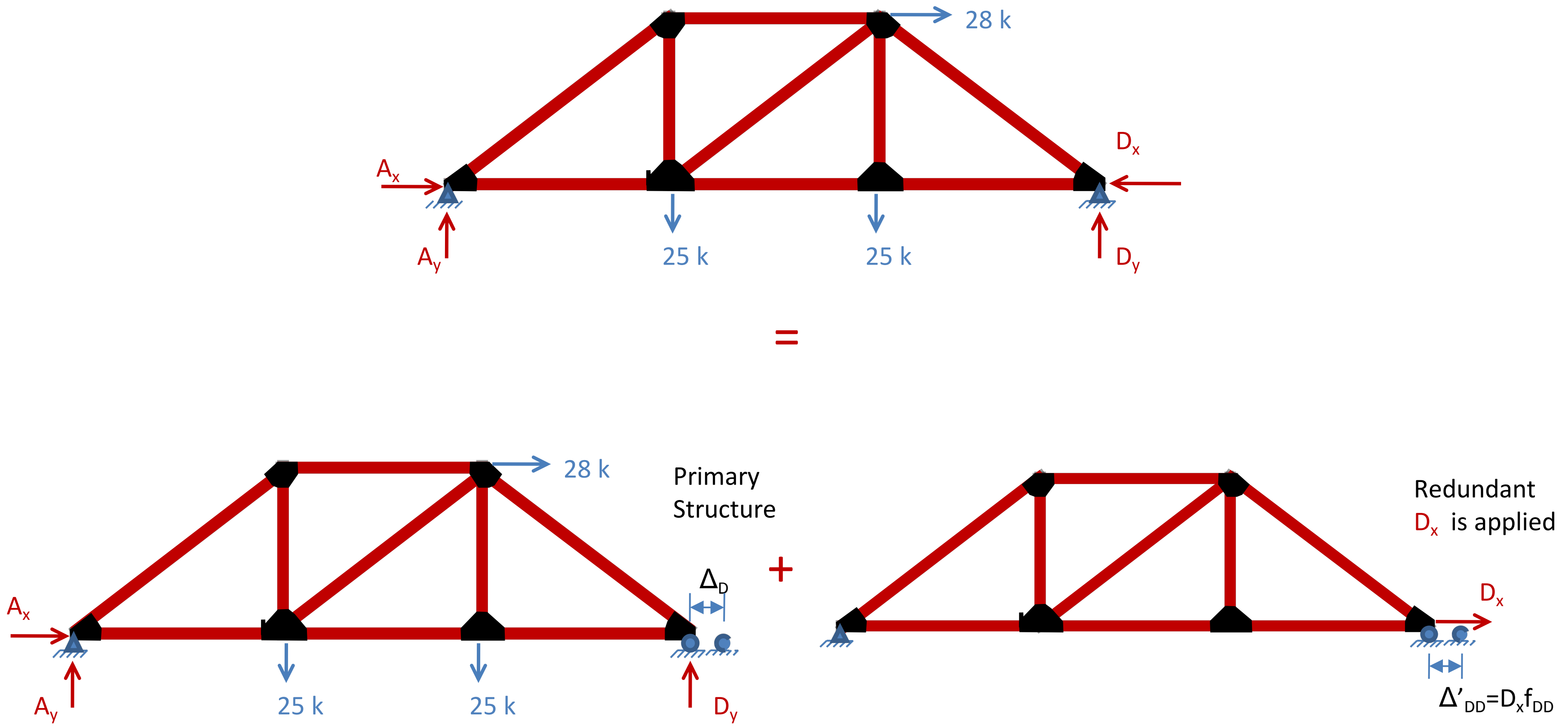
f_{DD} is horizontal deflection at point 'D' due to unit force.



Solution

Compatibility Equation

$$0 = \Delta_D + D_x f_{DD}$$



Solution

- We will use virtual work method to find Δ_D and f_{DD}.
- Deflection of truss is calculated by

$$\Delta_D = \sum \frac{nNL}{AE}$$

where

n = axial force in truss members due to virtual unit load acting at joint and in the direction of Δ_D

N = axial force in truss members due to real load acting that causes Δ_D

Solution

- We will use virtual work method to find Δ_D and f_{DD}.
- Deflection of truss is calculated by

$$f_{DD} = \sum \frac{n^2 L}{AE}$$

where

n = axial force in truss members due to real unit load acting at joint and in the direction of Δ_D

n = axial force in truss members due to virtual unit load acting at joint and in the direction of Δ_D

Solution

TABLE

Member	L (in.)	A (in.^{2})	N (k)	n (k)	nNL/A (k/in.)	n^2L/A	$F=N+nD_x$
AB	240	6	52	1	2,080	40	6.22
BC	240	6	42.67	1	1,706.8	40	-3.11
CD	240	6	42.67	1	1,706.8	40	-3.11
EF	240	6	-24	0	0	0	-24
BF	180	4	18	0	0	0	18
CF	180	4	25	0	0	0	25
AE	300	6	-30	0	0	0	-30
BF	300	4	11.67	0	0	0	11.67
DF	300	6	-53.33	0	0	0	-53.33
$\Delta_D = \sum \frac{nNL}{AE} = \frac{5{,}493.6\text{ k/in.}}{E}$					$\sum$ 5,493.6	120	

$$f_{DD} = \sum \frac{n^2 L}{AE} = \frac{120\,(1/\text{in.})}{E}$$

Solution

$$\Delta_D = \sum \frac{nNL}{AE}$$

$$\Delta_D = \frac{52 \times 1 \times 20 \times 12}{6E} + \frac{42.67 \times 1 \times 20 \times 12}{6E} + \frac{42.67 \times 1 \times 20 \times 12}{6E}$$

$$\Delta_D = \frac{2080}{6E} + \frac{1706.8}{6E} + \frac{1706.8}{6E}$$

$$\Delta_D = \frac{5493.6 \text{ k/in}}{E}$$

Solution

$$f_{DD} = \sum \frac{n^2 L}{AE}$$

$$f_{DD} = \frac{1\times1\times20\times12}{6E} + \frac{1\times1\times20\times12}{6E} + \frac{1\times1\times20\times12}{6E}$$

$$f_{DD} = \frac{120\,(1/\text{in})}{E}$$

Now put these results into Equation (1)

$$\frac{5493.6}{E} + D_x \times \frac{120}{E} = 0 \longrightarrow D_x = -45.78\,\text{k}\,(\leftarrow)$$

Solution

$$F = N + nD_x$$

$$F_{AB} = 52 + 1(-45.78) = 6.22 \text{ (T)}$$

$$F_{BC} = 42.67 + 1(-45.78) = -3.11 \text{ (C)}$$

$$F_{CD} = -3.11 \text{ (C)}$$

Equation of Equilibrium

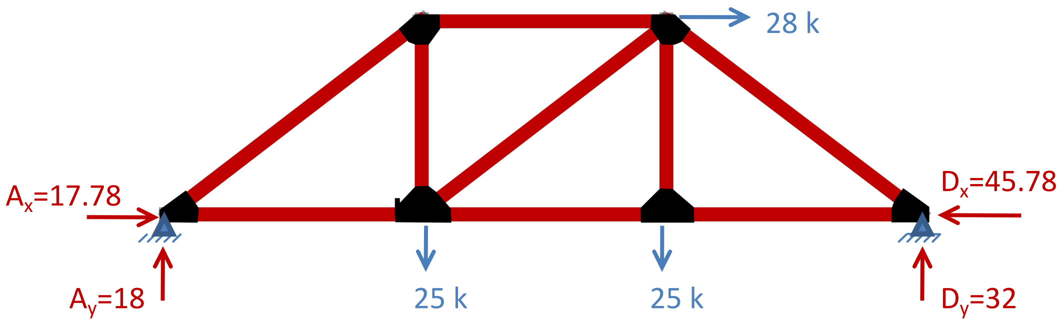
$$\sum F = 0$$

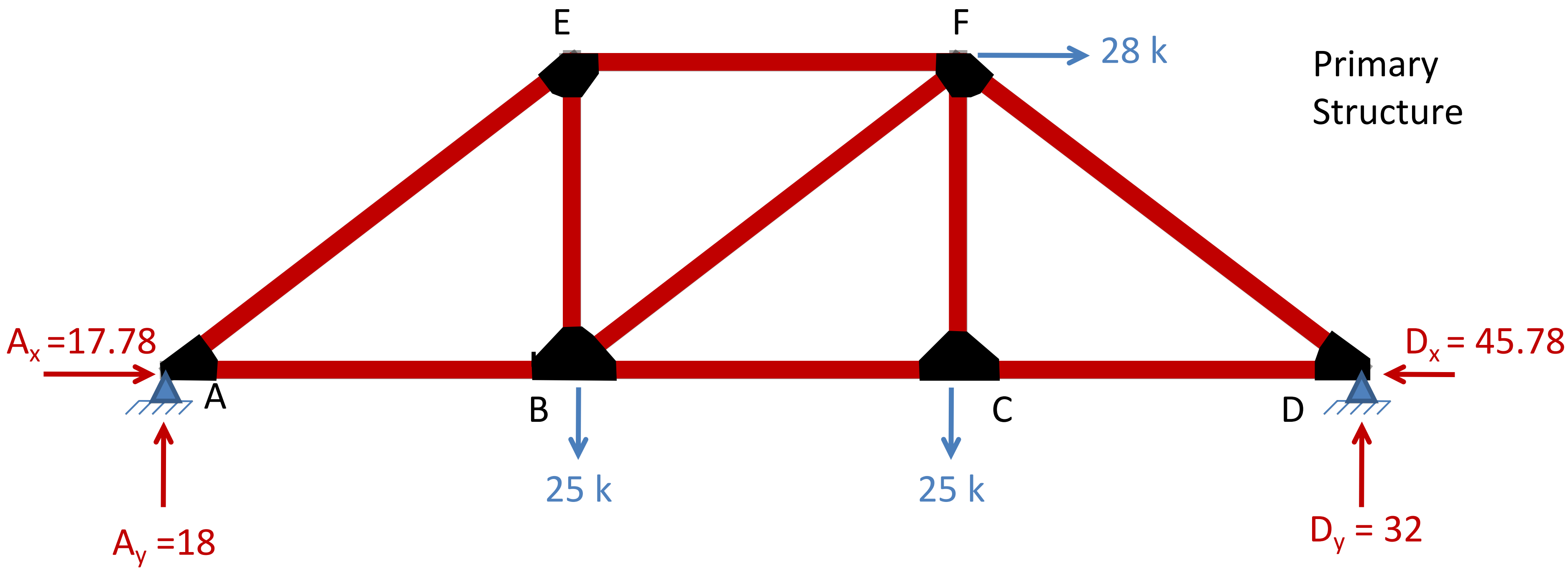
$$A_x + 28 - 45.78 = 0$$

$$A_x = 17.78 \text{ k} \quad (\rightarrow)$$

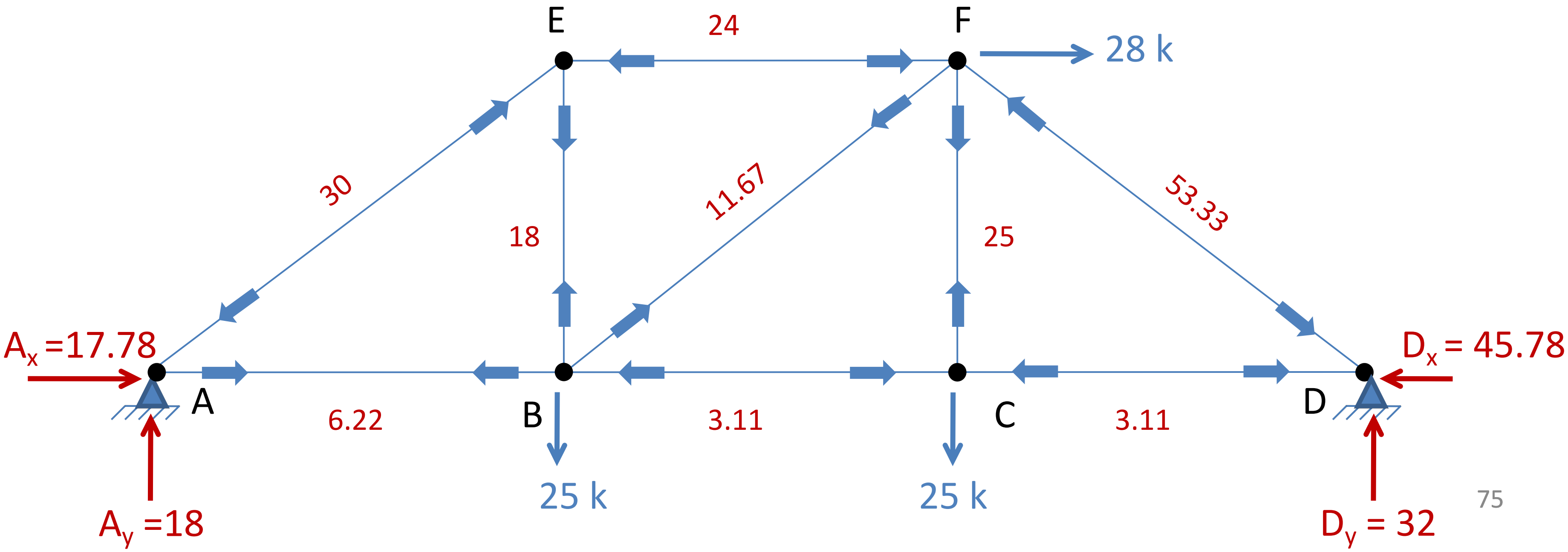
$$A_y = 18 \text{ k} \quad (\uparrow)$$

$$D_y = 32 \text{ k} \quad (\uparrow)$$



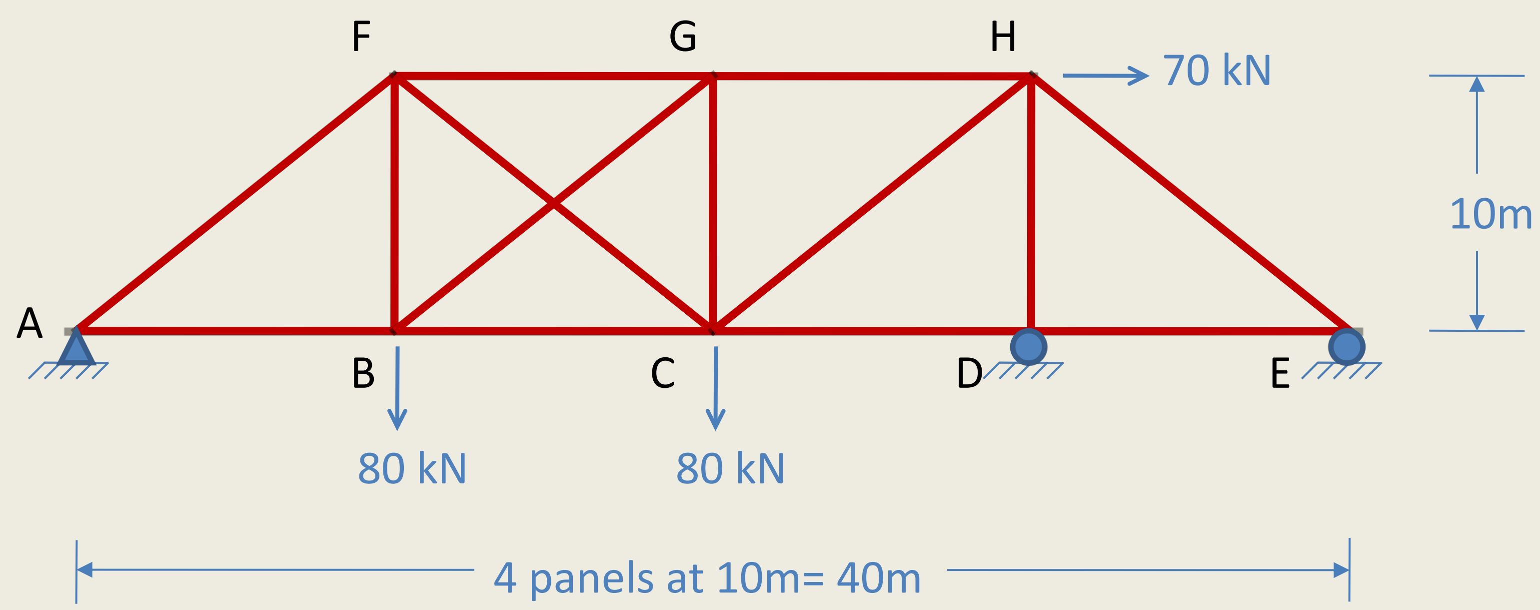


Primary Structure



Example 12

Determine the reactions and the force in each member of the truss shown in Fig. shown. EA = constant. E = 200 GPa., A = 4000 mm^2



Actual Truss

Solution

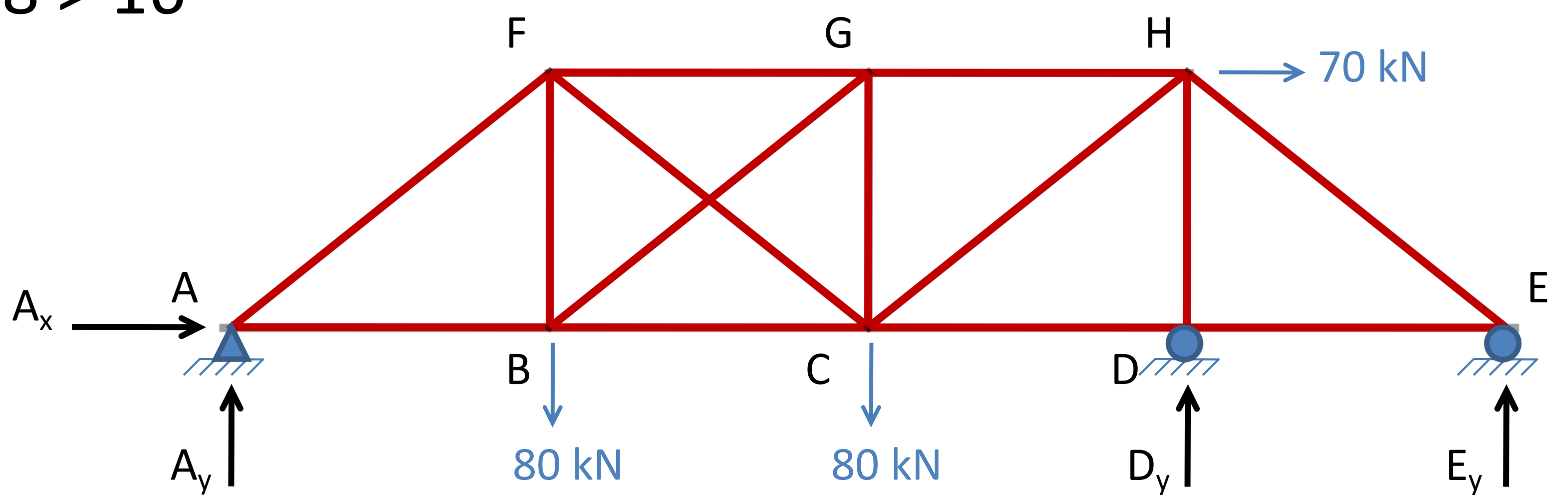
Principle of Superposition

Degree of Indeterminacy = 2

$b + r > 2j$

$14 + 4 > 2 \times 8$

$18 > 16$

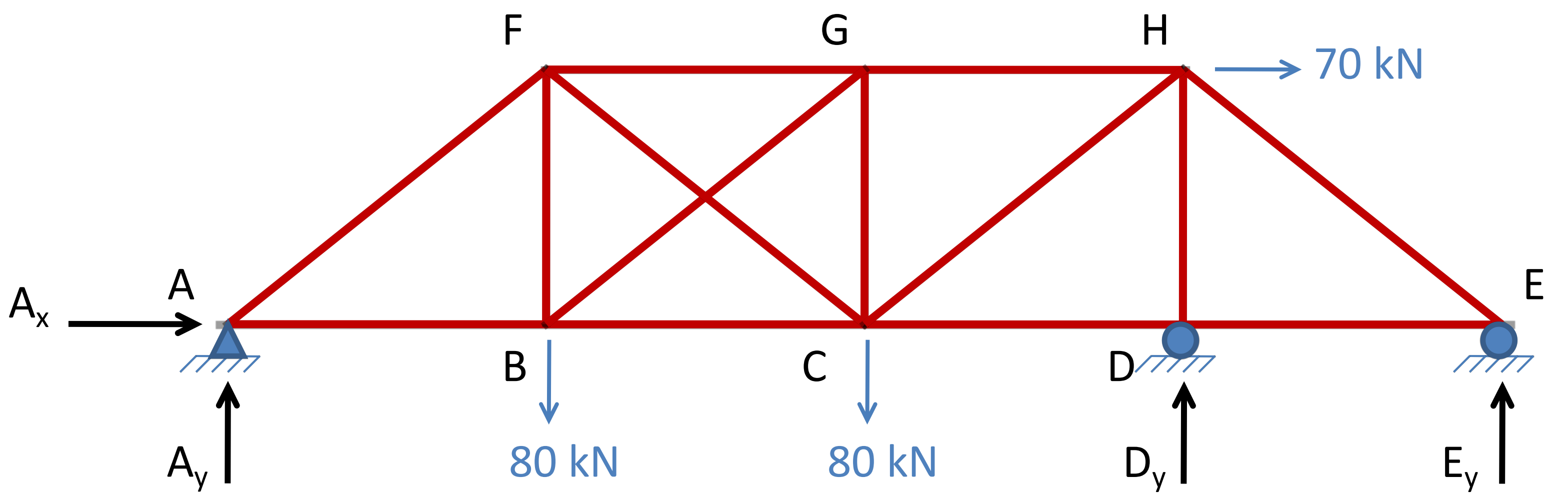


Actual Truss

Solution

Principle of Superposition

D_y at support D and force F_{BG} in member BG are selected as redundants.

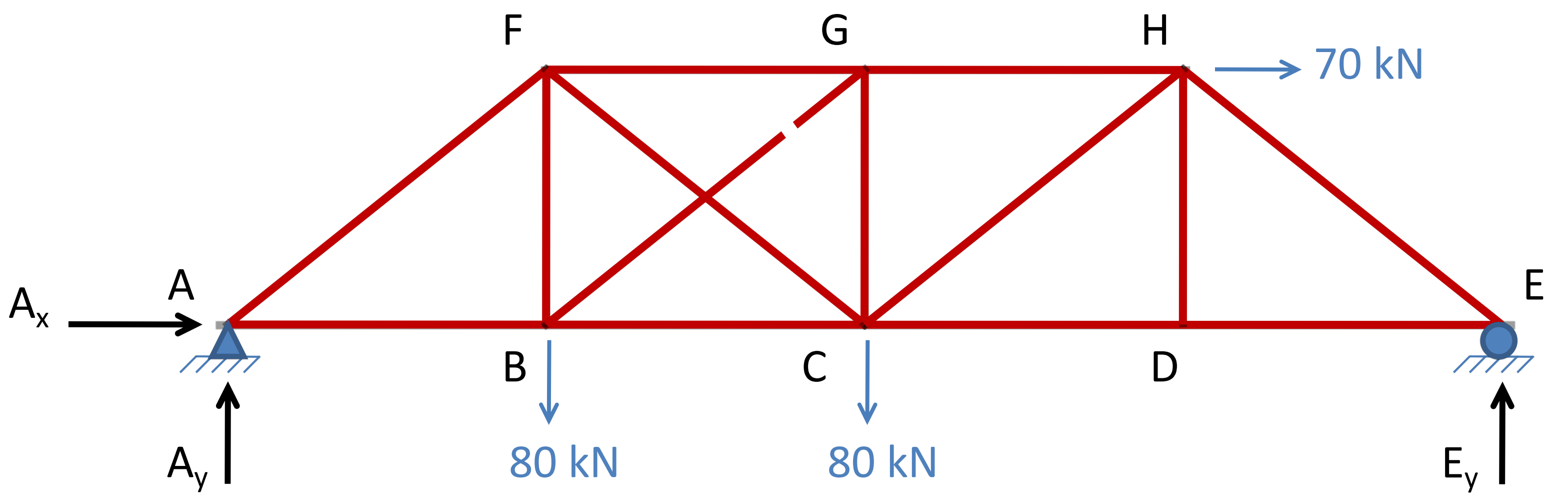


Actual Truss

Solution

Principle of Superposition

The roller support at 'D' is removed and member BG is cut to make the structure determinate.

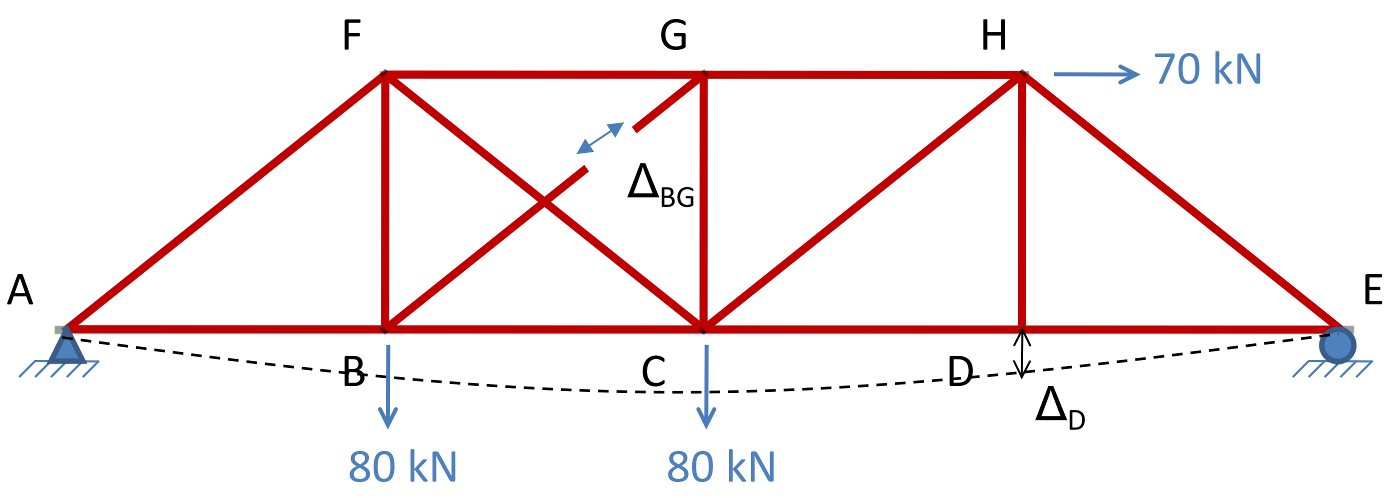


Determinate Truss

Solution

Principle of Superposition

This determinate truss is subjected separately to actual loading, redundant ‘ D_y ’ and redundant force in the redundant member BG.

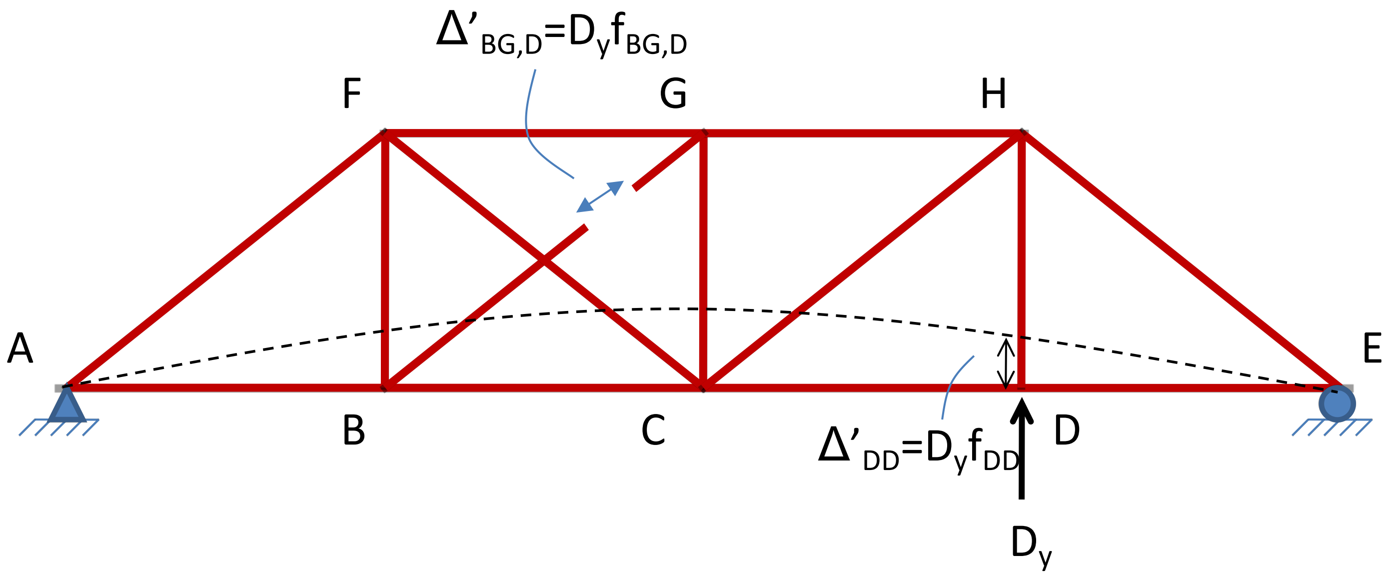


Primary structure subjected to actual loading

Solution

Principle of Superposition

This determinate truss is subjected separately to actual loading, redundant ‘ D_y ’ and redundant force in the redundant member BG.

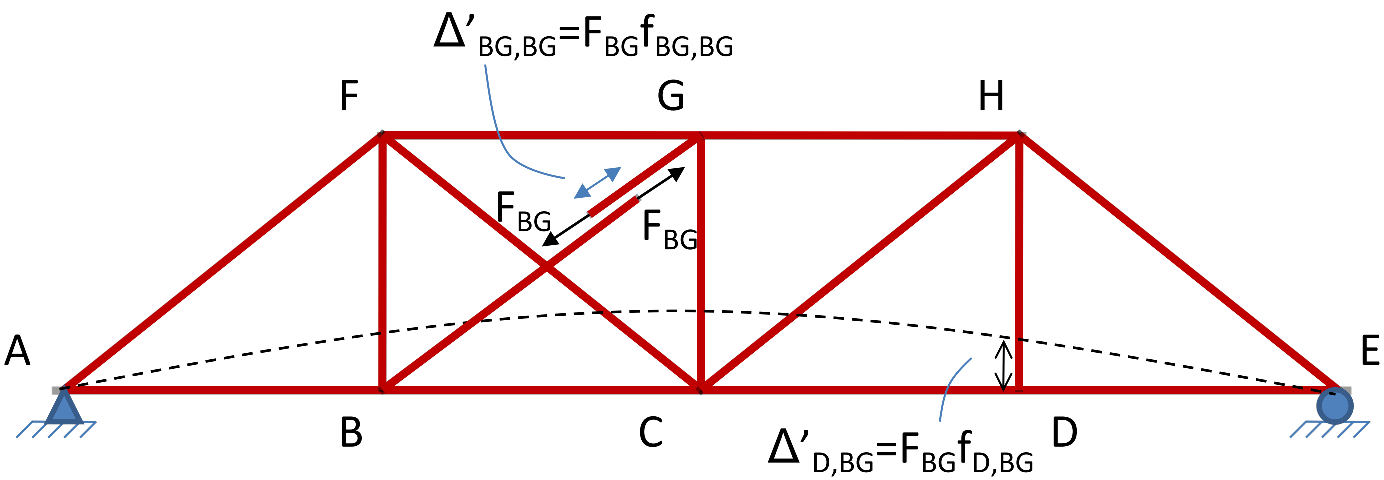


Redundant D_y applied

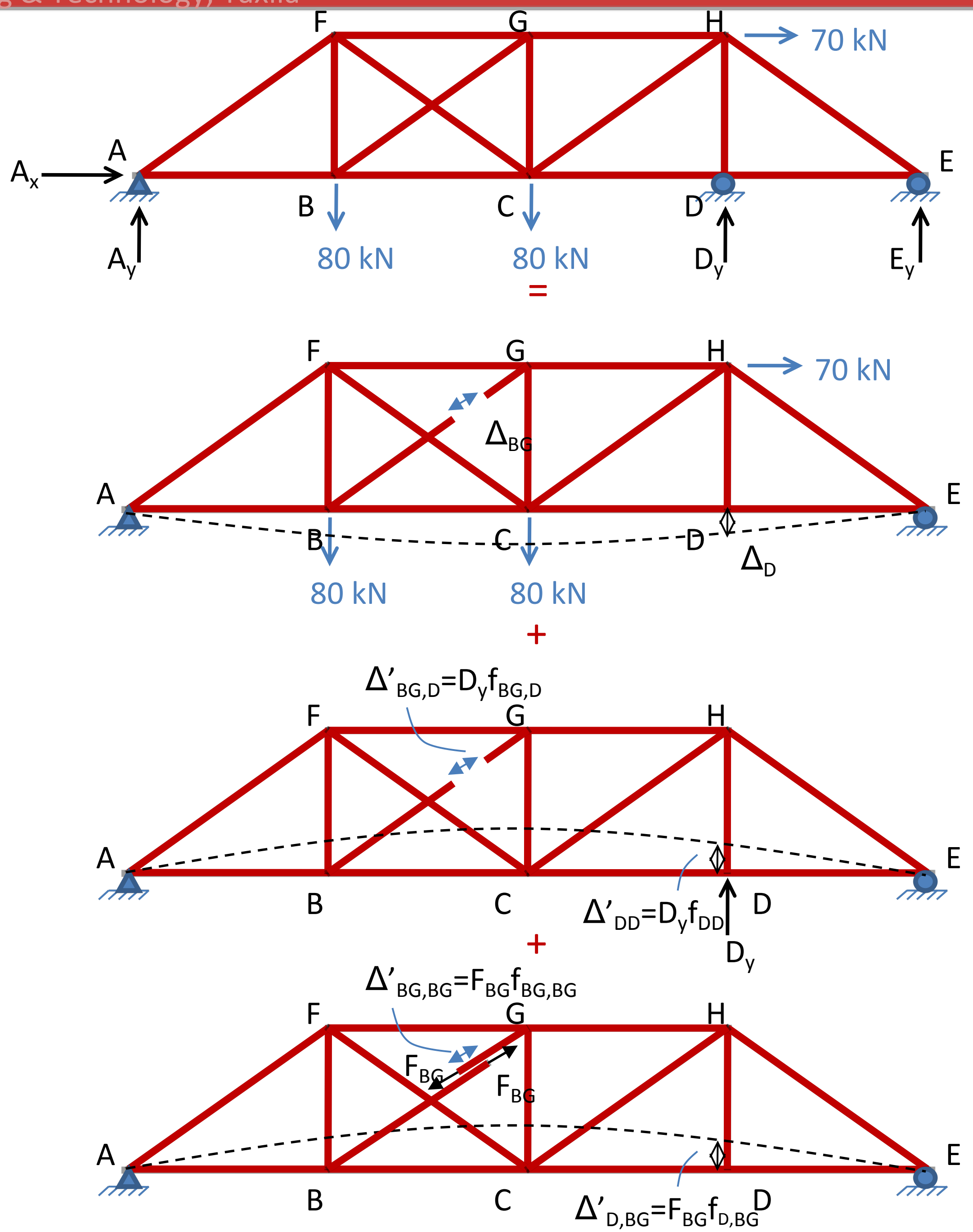
Solution

Principle of Superposition

This determinate truss is subjected separately to actual loading, redundant ‘ D_y ’ and redundant force in the redundant member BG.



Redundant F_{BG} applied



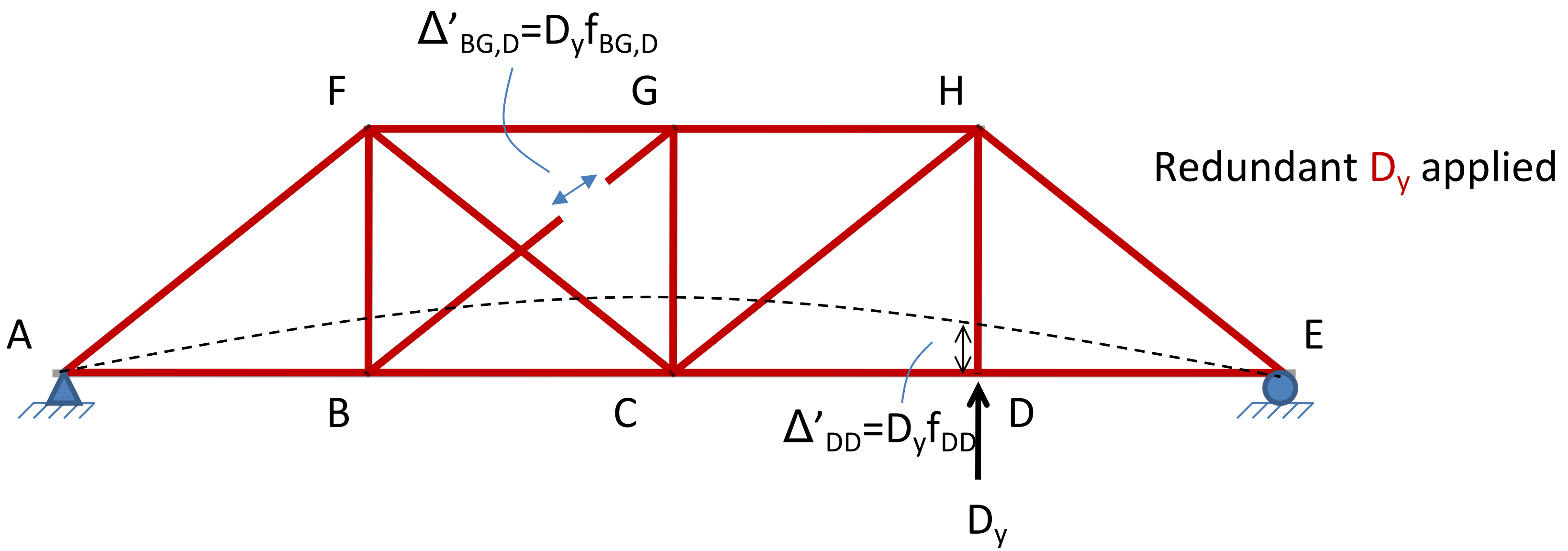
Actual Truss

Primary structure

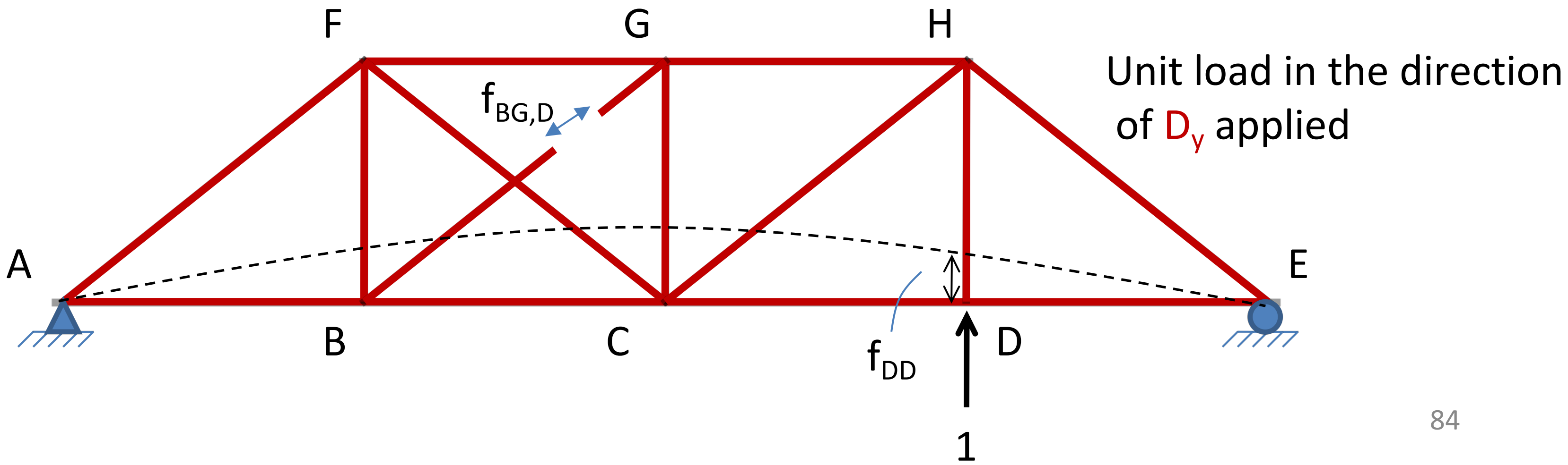
Redundant D_y applied

Redundant F_{BG} applied

Solution

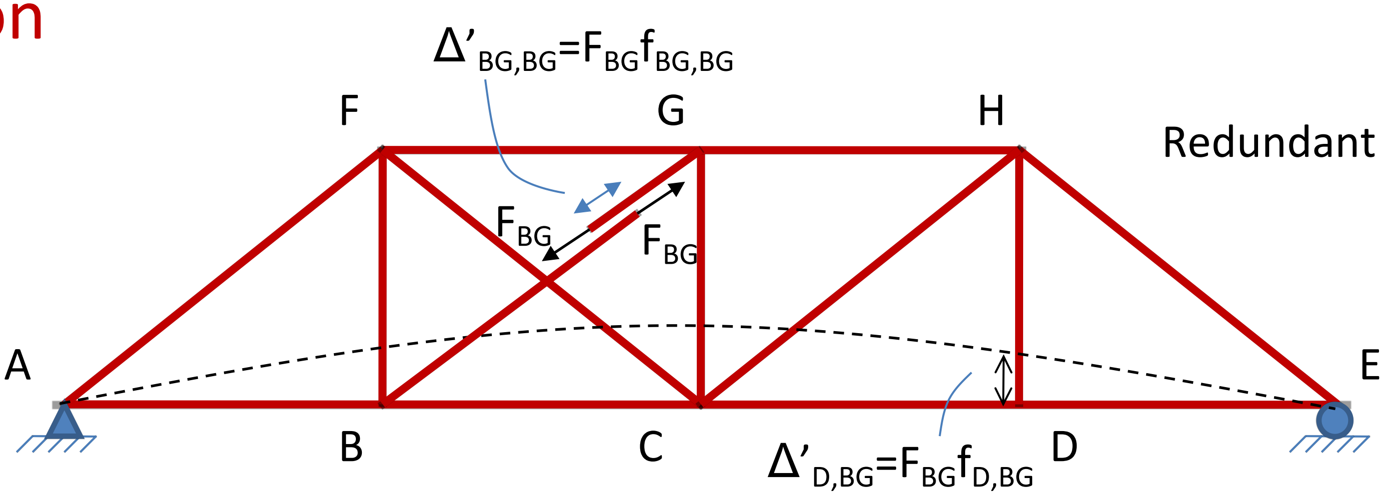


Δ’BG,D=DyfBG,D
F
G
H
Redundant Dy applied
A
E
B
C
Δ’DD=DyfDD
D
Dy

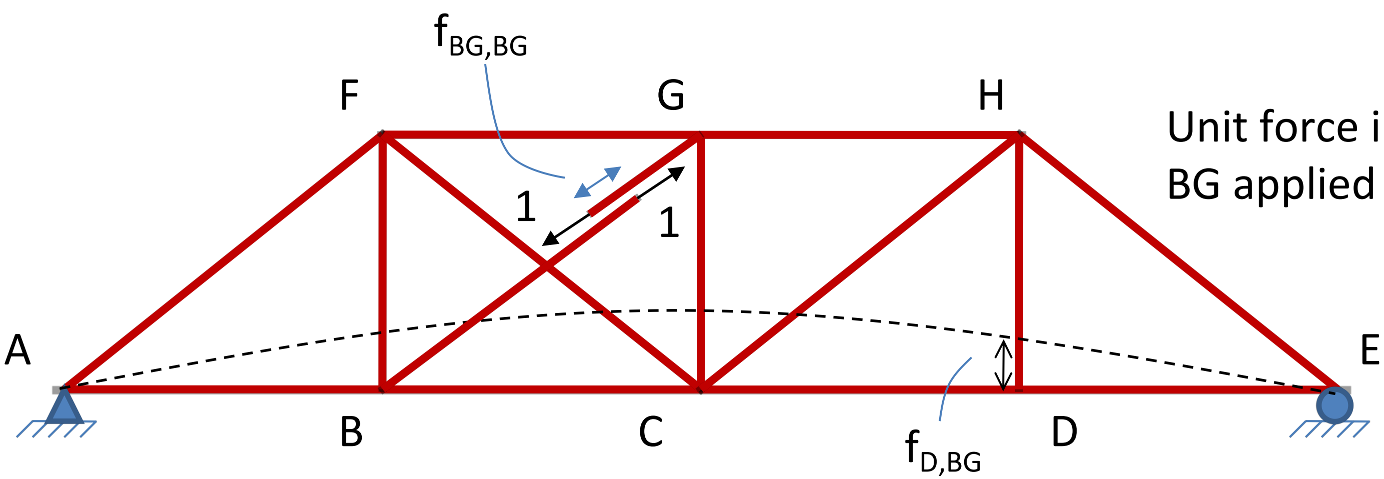


F
G
H
Unit load in the direction of Dy applied
fBG,D
A
E
B
C
fDD
D
1

Solution

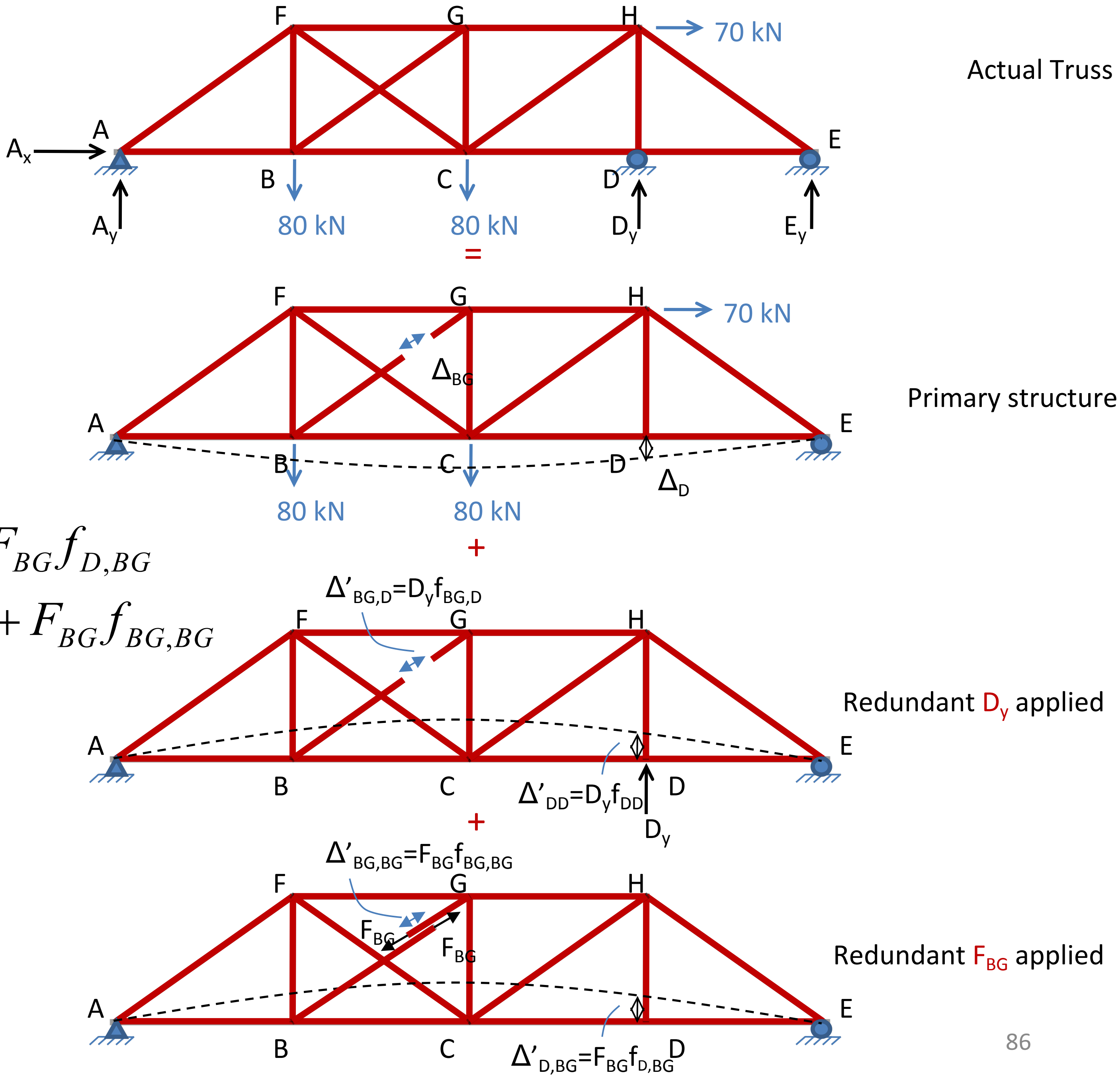


Redundant F_{BG} applied



Unit force in member BG applied

Compatibility Equation



$$0 = \Delta_D + D_y f_{DD} + F_{BG} f_{D,BG}$$

$$0 = \Delta_{BG} + D_y f_{BG,D} + F_{BG} f_{BG,BG}$$

Compatibility Equation

$$0 = \Delta_D + D_y f_{DD} + F_{BG} f_{D,BG}$$

$$0 = \Delta_{BG} + D_y f_{BG,D} + F_{BG} f_{BG,BG}$$

Δ_D = vertical deflection at joint D of primary truss due to external loading

Δ_{BG} = relative displacement b/w cutting ends of member BG due to external loading

f_{DD} = vertical deflection at joint D due to a unit load at joint D

$f_{BG,D}$ = relative displacement b/w cutting ends of member BG due to unit load at D

$f_{BG,BG}$ = relative displacement b/w cutting ends of member BG due to unit force

$f_{D,BG}$ = vertical deflection at joint D due to a unit force in member BG

Compatibility Equation

We will use the method of virtual work to find the deflections

$$0 = \Delta_D + D_y f_{DD} + F_{BG} f_{D,BG}$$

$$0 = \Delta_{BG} + D_y f_{BG,D} + F_{BG} f_{BG,BG}$$

$$\Delta_D = \sum \frac{N n_D L}{AE} \qquad f_{DD} = \sum \frac{n_D n_D L}{AE} \qquad f_{D,BG} = \sum \frac{n_D n_{BG} L}{AE}$$

$$\Delta_{BG} = \sum \frac{N n_{BG} L}{AE} \qquad f_{BG} = \sum \frac{n_{BG} n_{BG} L}{AE} \qquad f_{BG,D} = \sum \frac{n_{BG} n_D L}{AE}$$

Compatibility Equation

$$\Delta_D = \sum \frac{N n_D L}{AE} \qquad f_{DD} = \sum \frac{n_D n_D L}{AE} \qquad f_{D,BG} = \sum \frac{n_D n_{BG} L}{AE}$$

$$\Delta_{BG} = \sum \frac{N n_{BG} L}{AE} \qquad f_{BG} = \sum \frac{n_{BG} n_{BG} L}{AE} \qquad f_{BG,D} = \sum \frac{n_{BG} n_D L}{AE}$$

N = member forces due to external loading

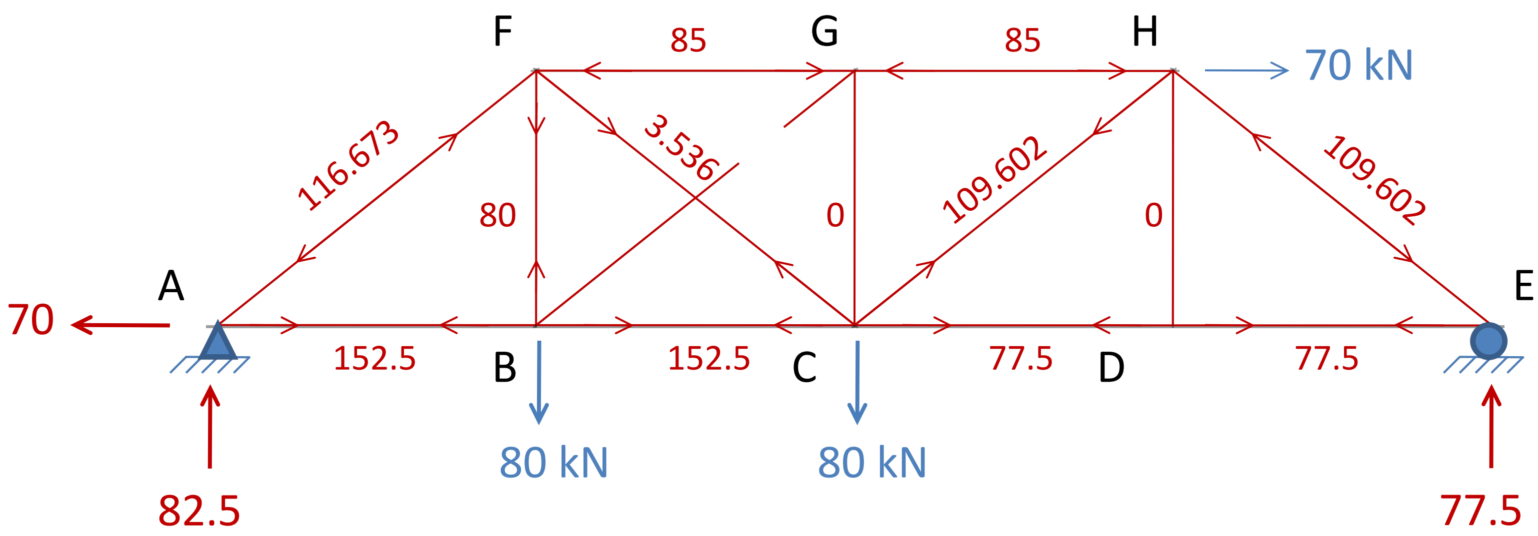
n_D = member forces due to unit load at joint D

n_{BG} = member forces due to unit force in member BG

The numerical values of the member forces, as computed by the method of joints, are shown in next figures, and are tabulated in the TABLE

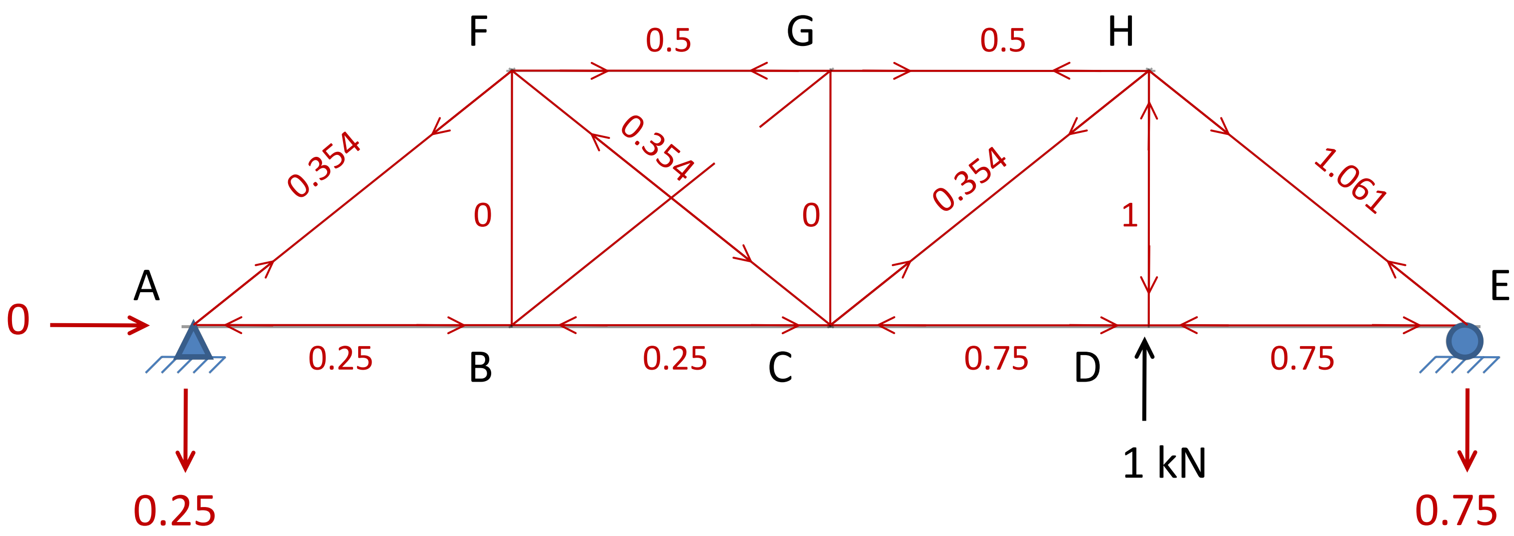
Solution

N = member forces due to external loading



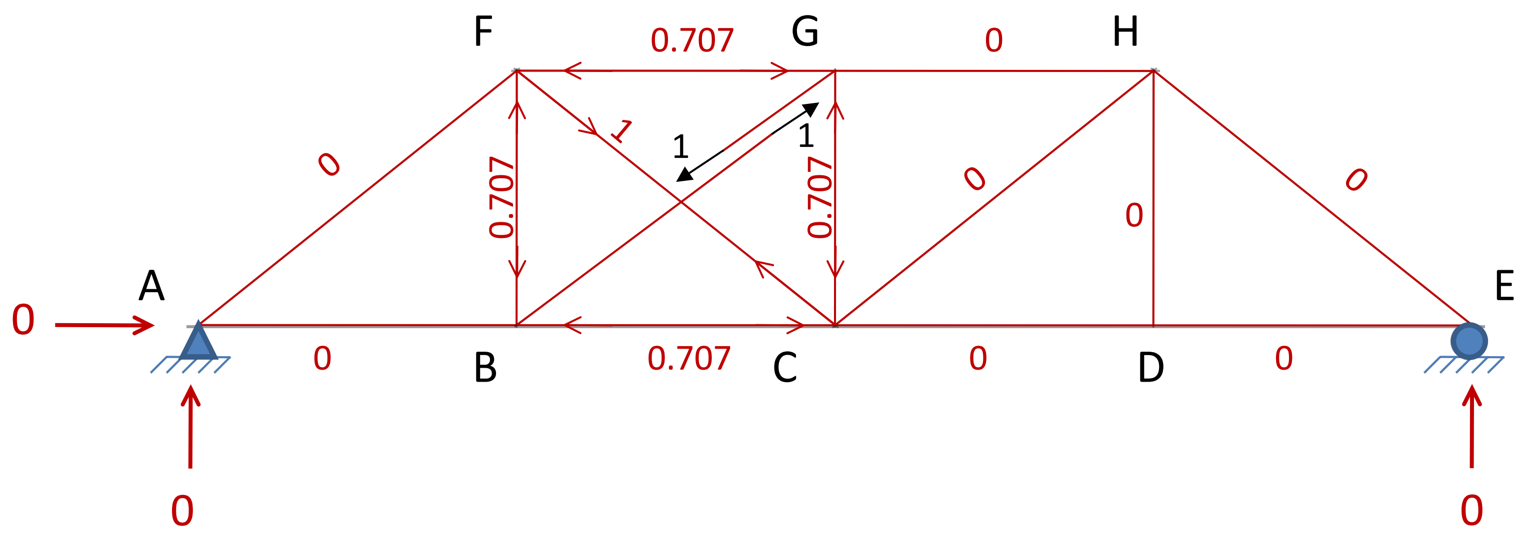
Solution

n_D = member forces due to unit load at joint D



Solution

n_{BG} = member forces due to unit force in member *BG*



Member	L (m)	N (kN)	n_D (kN/kN)	n_{BG} (kN/kN)	Nn_DL (kN.m)	$Nn_{BG}L$ (kN.m)	n_D^2L (m)	n_{BG}^2L (m)	$n_Dn_{BG}L$ (m)	$F = N + n_DD_y + n_{BG}F_{BG}$ (kN)
AB	10	152.5	-0.25	0	-381.25	0	0.625	0	0	128.373
BC	10	152.5	-0.25	-0.707	-381.25	-1078.175	0.625	5	1.768	104.265
CD	10	77.5	-0.75	0	-581.25	0	5.625	0	0	5.12
DE	10	77.5	-0.75	0	-581.25	0	5.625	0	0	5.12
FG	10	-85	0.5	-0.707	-425	600.95	2.5	5	-3.535	-60.855
GH	10	-85	0.5	0	-425	0	2.5	0	0	-36.747
BF	10	80	0	-0.707	0	-565.60	0	5	0	55.891
CG	10	0	0	-0.707	0	0	0	5	0	-24.109
DH	10	0	-1	0	0	0	10	0	0	-96.507
AF	14.142	-116.673	0.354	0	-584.096	0	1.772	0	0	-82.51
BG	14.142	0	0	1	0	0	0	14.142	0	34.1
CF	14.142	3.536	-0.354	1	-17.702	50.006	1.772	14.142	-5.006	3.473
CH	14.142	109.602	0.354	0	548.697	0	1.772	0	0	143.765
EH	14.142	-109.602	1.061	0	-1644.541	0	15.92	0	0	-7.208
				$\sum$	-4472.642	-992.819	48.736	48.284	-6.773	

Compatibility Equation

$$0 = \Delta_D + D_y f_{DD} + F_{BG} f_{D,BG}$$

$$0 = \Delta_{BG} + D_y f_{BG,D} + F_{BG} f_{BG,BG}$$

$$\Delta_D = -\frac{4{,}472.642\, kN.m}{AE}$$

$$f_{BG,BG} = \frac{48.284\, m}{AE}$$

$$\Delta_{BG} = -\frac{992.819\, kN.m}{AE}$$

$$f_{BG,D} = f_{D,BG} = -\frac{6.773\, m}{AE}$$

$$f_{DD} = \frac{48.736\, m}{AE}$$

By substituting these values into the above equations

Compatibility Equation

$$-4{,}472.642 + 48.736D_y - 6.773F_{BG} = 0$$

$$-992.819 - 6.773D_y + 48.284F_{BG} = 0$$

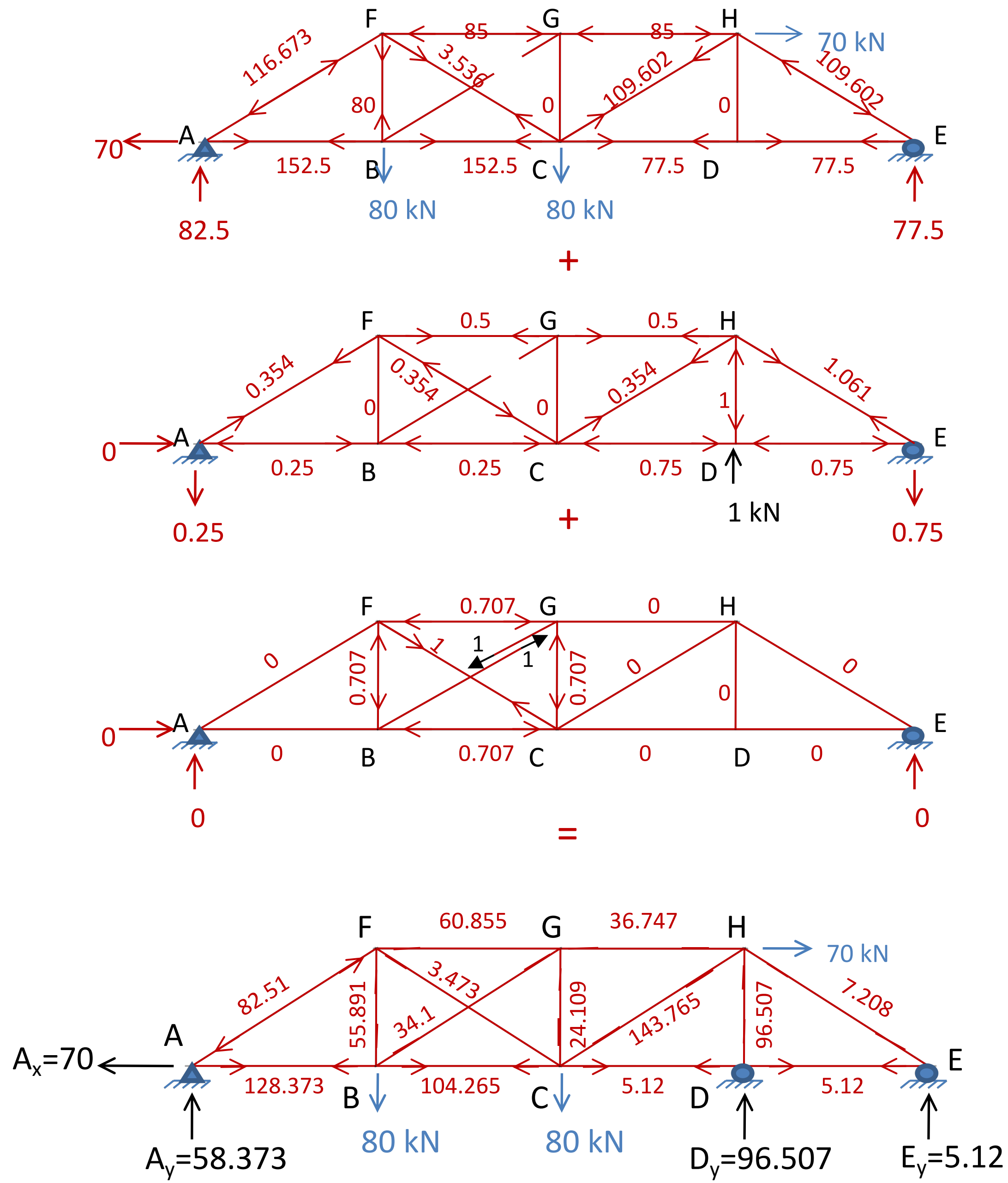
Solving these equations simultaneously for D_y and F_{BG}

$$D_y = 96.507\,kN \uparrow$$

$$F_{BG} = 34.1\,kN$$

The remaining reactions of the indeterminate truss can now be determined by superposition of reactions of primary truss due to the external loading and due to each of the redundants.

The forces in the remaining members of the indeterminate truss can be determined by using the superposition relationship



Actual Truss

Actual Truss