MATRIX FLEXIBILITY METHOD

Flexibility method

- Also referred to as the ***force*** *or* ***compatibility*** *method*

- *It is essentially a generalization* in matrix form of the classical method of ***consistent deformations***.

- In this approach, the primary unknowns are the ***redundant forces***, which are calculated first by solving the structure's ***compatibility equation***. Once the redundant forces are known, the ***displacements*** can be evaluated by applying the **equations of equilibrium** and the appropriate ***member force–displacement relations***.

Compatibility Equations

A stable structure apart from satisfying equilibrium conditions should also satisfy all the compatibility conditions. These conditions require that the displacements and rotations be continuous throughout the structure and compatible with the nature supports conditions.

For example, at a fixed support this requires that displacement and slope should be zero.

FLEXIBILITY,FLEXIBILITY COEFFICIENT AND FLEXIBILITY MATRIX

Flexibility:

Flexibility of a member is defined as deformation produced by a unit load

If a member which behave elastically is subjected to varying axial load (W) as shown in fig.1 and a graph is drawn of load (W) versus displacement (Δ) the result will be a straight line as shown in fig.2, the slop of this line is called flexibility.

Mathematically it can be expressed as

$$F=\Delta/W$$

In other words Flexibility “F” is displacement produced by unit force

$$\Delta=F.W$$

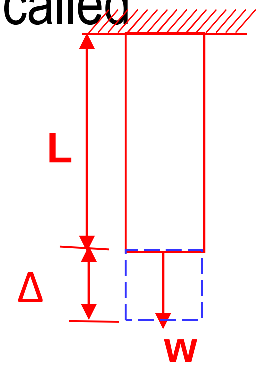


Fig.1

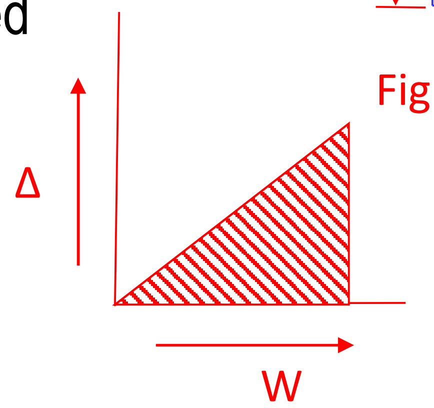
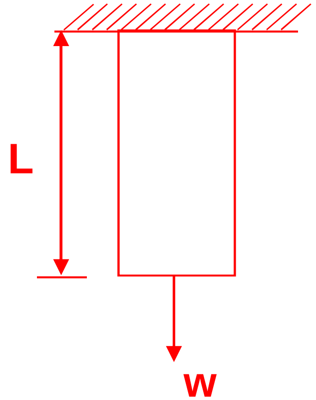


Fig.2

From stress strain relationship

$$\Delta = \frac{w\,L}{AE}$$



Where;

W = Load or Unit Load

$$\Delta = \frac{L}{AE} \cdot 1$$

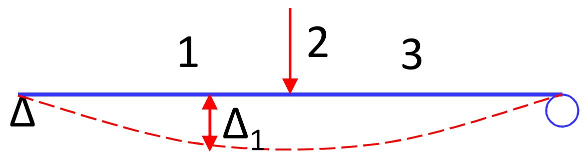
Where; $\frac{L}{AE}$ is flexibility or flexibility co-efficient that deformation produced by unit load and is denoted by "F".

$$\Delta = F \cdot w$$

In case of Beam element

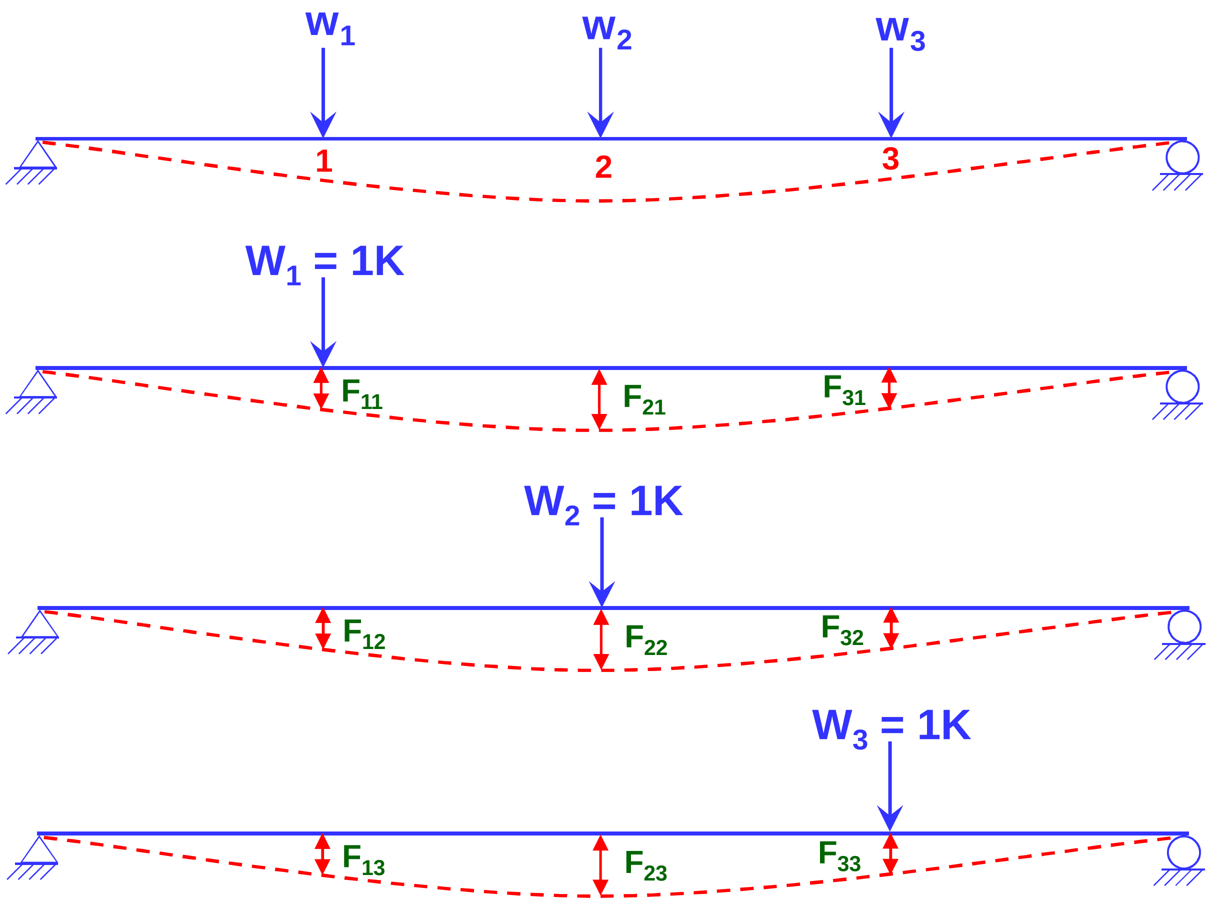
Let us consider a beam along which we have chosen three points designated as 1,2 and 3.if we apply a load W_2 at point 2,the deflection Δ_1 at point 1 can be expressed in the form

$$\Delta_1 = F_{12} . W_2$$



The term F_{12} is a flexibility coefficient.
Where F_{ab} indicates deformation at point/location “a” due to load/Moment at location “b”.

If instead of a single load at 2,we apply loads at all three points ,the deflection at 1,2 and 3 will be given by



$$\Delta_1 = F_{11}w_1 + F_{12}w_2 + F_{13}w_3$$

$$\Delta_2 = F_{21}w_1 + F_{22}w_2 + F_{23}w_3$$

$$\Delta_3 = F_{31}w_1 + F_{32}w_2 + F_{33}w_3$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

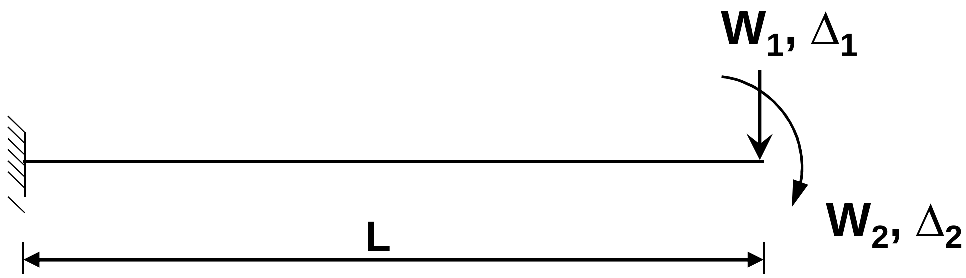
$$[\Delta] = [F]\,[W]$$

Δ's are called Structure Deformations.

w's are called Structure Loads.

F's are called Structure Flexibility coefficients.

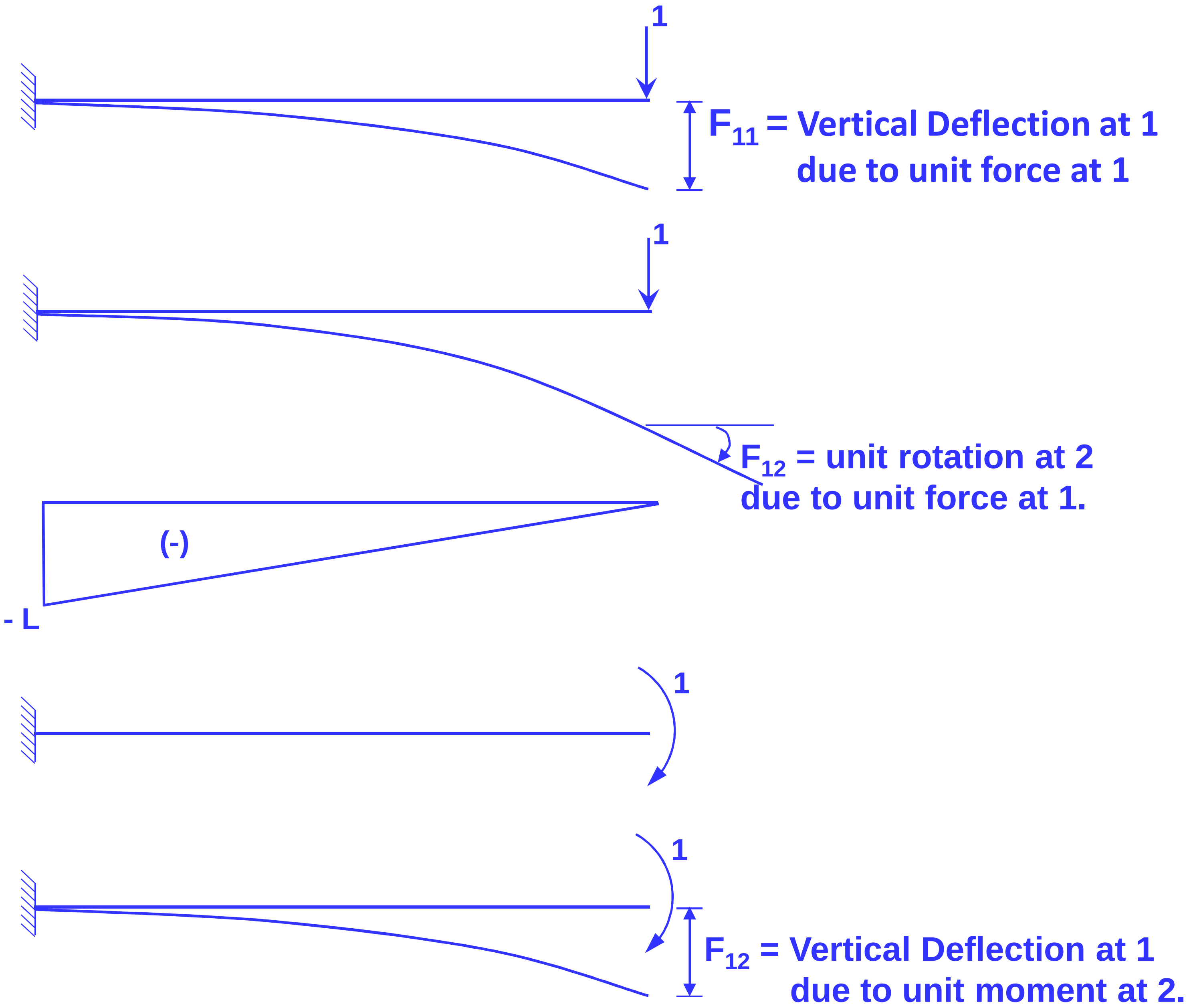
The Matrix [F] that contain flexibility coefficients and relates the deflection [Δ] to the loads [W] is referred to as flexibility matrix.

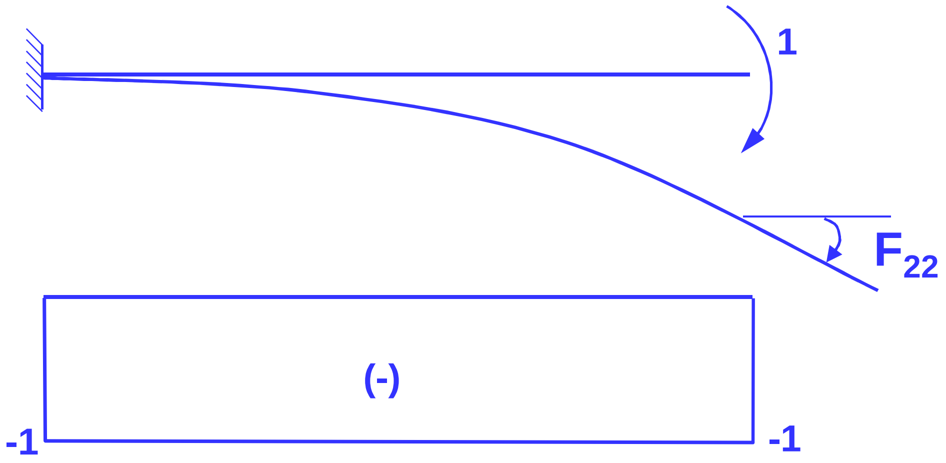


$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix}$$

First Column of flexibility matrix is obtained by applying unit load/Moment at location “1”.

Similarly 2nd Column of flexibility matrix can be obtained by applying unit load/Moment at location “2” and obtaining deformation at location 1 and 2.


1
F11 = Vertical Deflection at 1
due to unit force at 1
1
F12 = unit rotation at 2
due to unit force at 1.
(-)
- L
1
1
F12 = Vertical Deflection at 1
due to unit moment at 2.



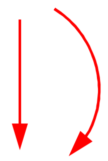
F_{22} = Unit rotation at 2 due to unit moment at 2

Using moment area method , we obtained

$$F_{11} = \frac{L^3}{3EI} \qquad F_{21} = \frac{L^2}{2EI}$$

$$F_{12} = \frac{L^2}{2EI} \qquad F_{22} = \frac{L}{EI}$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \begin{pmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ \frac{L^2}{2EI} & \frac{L}{EI} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$



+ve sign convention