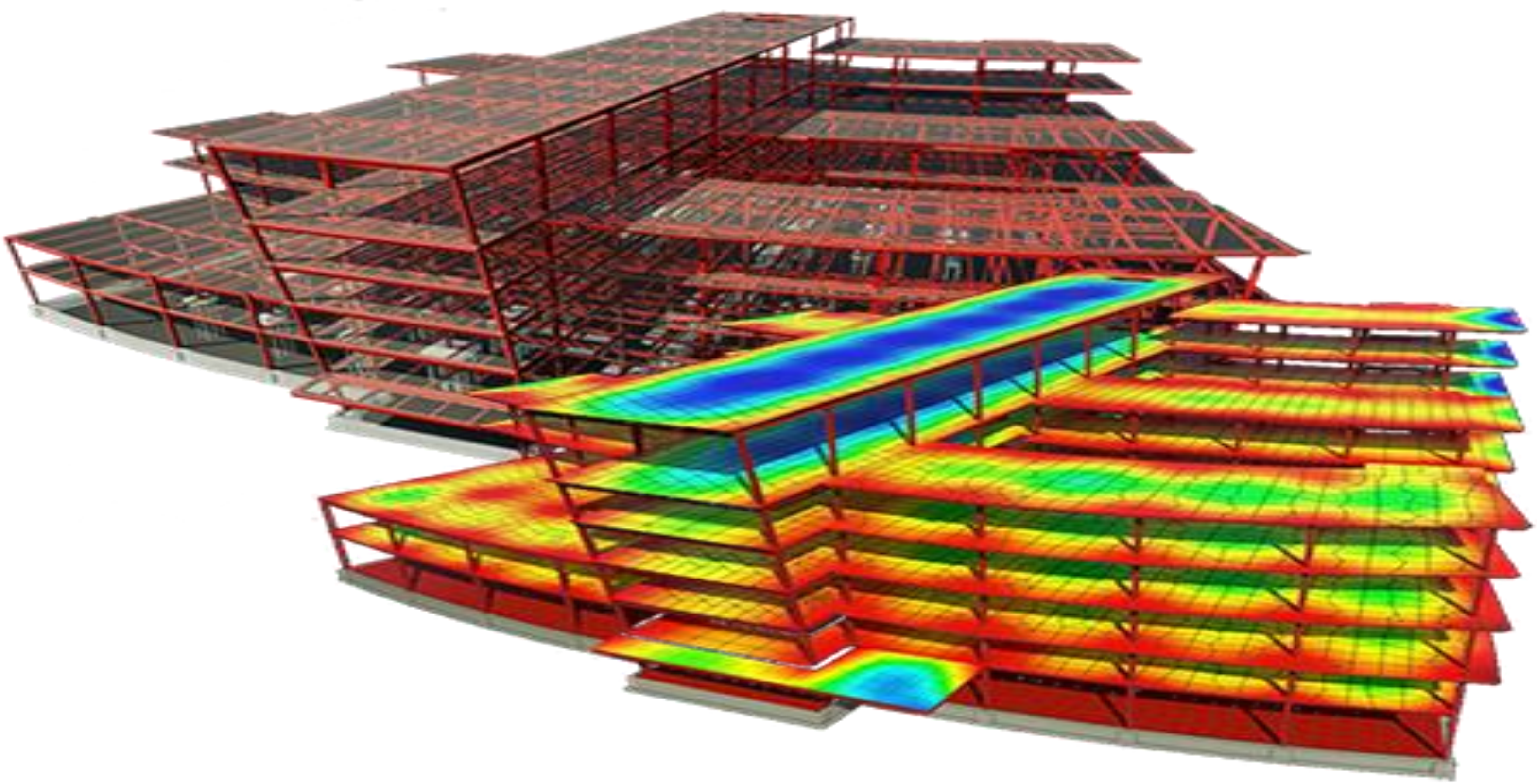
Advanced Structural Analysis

INTRODUCTION:

The structural analysis is a mathematical process by which the response of a structure to specified loads is determined. This response is measured by determining the internal forces or stresses and displacements or deformations throughout the structure.

The results of the analysis are used to verify a structure's fitness for use,



Methods of Analysis

Methods of Structural Analysis can be broadly classified into two categories, namely, the force (flexibility) methods and the displacement (stiffness) methods, depending on the type of unknowns (forces or displacements, respectively)

Force or Flexibility Method

In this method, redundant forces are treated as unknown. These are then related to corresponding displacements.

Methods

Classical Method

- Castiglione's 2nd theorem
- Consistent Deformation
- Method of Least Work
- Three Moment Equation

Modern Method

- Matrix Flexibility Method

Displacement or Stiffness Method of Analysis

In this method, the rotation or the nodal displacements are treated as unknown. These are then related to corresponding forces

Classical Method

- Slop Deflection Method
- Moment Distribution Method
- Kani's Method

Modern Method

- Matrix Stiffness Method

Matrix Analysis of Structures

Matrix analysis of a structure is a branch of structural analysis in which matrix algebraic is used as a tool for the analysis of structure.

- **Matrix Flexibility Method**
- **Matrix Stiffness Method**

Systematic - Conveniently programming.

General - Same overall format of analytical procedure can be applied to various types of framed structures. For example, a computer program developed to analyze plane trusses can be modified with relative ease to analyze space trusses or frames.

As the analysis of large and highly redundant structures by classical methods can be quite ***time consuming***, matrix methods are commonly used.
However, classical methods are still preferred by many engineers for ***analyzing smaller structures***.

Continue…

To study matrix methods there are some pre-requirements :

i) Matrix Algebra - Addition, subtraction ,Multiplication & inversion of matrices.

ii) Methods of finding out Displacements i.e. slope & deflection at any point in a structure, such as
a)Unit load method or Strain energy method
b) Moment area method etc.

iii) Study of Indeterminacies – Static indeterminacy & kinematic indeterminacy

MATRIX ALGEBRA

A *matrix is defined as a rectangular array of quantities arranged in rows and columns. A matrix with m rows and n columns can be expressed as:-*

$$
\begin{array}{rcl}
\mathbf{A} = [A] = & \begin{bmatrix}
A_{11} & A_{12} & A_{13} & \cdots & & \cdots & A_{1n} \\
A_{21} & A_{22} & A_{23} & \cdots & & \cdots & A_{2n} \\
A_{31} & A_{32} & A_{33} & \cdots & & \cdots & A_{3n} \\
\cdots & \cdots & \cdots & \cdots & A_{ij} & \cdots & \\
A_{m1} & A_{m2} & A_{m3} & \cdots & & \cdots & A_{mn}
\end{bmatrix} & \begin{array}{l} \\ \\ \\ i\text{th row} \\ \\ \end{array} \\
& \qquad\qquad\qquad\qquad j\text{th column} \qquad\quad m \times n &
\end{array}
$$

MATRIX ALGEBRA

Column Matrix (Vector)

If all the elements of a matrix are arranged in a single column (i.e., $n = 1$), it is called a *column matrix*. Column matrices are usually referred to as *vectors*, and are sometimes denoted by italic letters enclosed within braces. An example of a column matrix or vector is given by

$$\mathbf{B} = \{B\} = \begin{bmatrix} 35 \\ 9 \\ 12 \\ 3 \\ 26 \end{bmatrix}$$

Row Matrix

A matrix with all of its elements arranged in a single row (i.e., $m = 1$) is referred to as a *row matrix*. For example,

$$\mathbf{C} = [9 \quad 35 \quad -12 \quad 7 \quad 22]$$

MATRIX ALGEBRA

Square Matrix

If a matrix has the same number of rows and columns (i.e., $m = n$), it is called a *square matrix*. An example of a 4×4 square matrix is given by

$$\mathbf{A} = \begin{bmatrix} 6 & 12 & 0 & 20 \\ 15 & -9 & -37 & 3 \\ -24 & 13 & 8 & 1 \\ 40 & 0 & 11 & -5 \end{bmatrix} \tag{2.2}$$

main diagonal

MATRIX ALGEBRA

Symmetric Matrix

When the elements of a square matrix are symmetric about its main diagonal (i.e., $A_{ij} = A_{ji}$), it is termed a *symmetric matrix*. For example,

$$\mathbf{A} = \begin{bmatrix} 6 & 15 & -24 & 40 \\ 15 & -9 & 13 & 0 \\ -24 & 13 & 8 & 11 \\ 40 & 0 & 11 & -5 \end{bmatrix}$$

MATRIX ALGEBRA

Lower Triangular Matrix

If all the elements of a square matrix above its main diagonal are zero, (i.e., $A_{ij} = 0$ for $j > i$), it is referred to as a *lower triangular matrix*. An example of a 4×4 lower triangular matrix is given by

$$\mathbf{A} = \begin{bmatrix} 8 & 0 & 0 & 0 \\ 12 & -9 & 0 & 0 \\ 33 & 17 & 6 & 0 \\ -2 & 5 & 15 & 3 \end{bmatrix}$$

Upper Triangular Matrix

When all the elements of a square matrix below its main diagonal are zero (i.e., $A_{ij} = 0$ for $j < i$), it is called an *upper triangular matrix*. An example of a 3×3 upper triangular matrix is given by

$$\mathbf{A} = \begin{bmatrix} -7 & 6 & 17 \\ 0 & 12 & 11 \\ 0 & 0 & 20 \end{bmatrix}$$

MATRIX ALGEBRA

Diagonal Matrix

A square matrix with all of its off-diagonal elements equal to zero (i.e., $A_{ij} = 0$ for $i \neq j$), is called a *diagonal matrix*. For example,

$$\mathbf{A} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 27 \end{bmatrix}$$

MATRIX ALGEBRA

Unit or Identity Matrix

If all the diagonal elements of a diagonal matrix are equal to 1 (i.e., $I_{ij} = 1$ and $I_{ij} = 0$ for $i \neq j$), it is referred to as a *unit* (or *identity*) *matrix*. Unit matrices are commonly denoted by $\mathbf{I}$ or $[I]$. An example of a 3×3 unit matrix is given by

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Null Matrix

If all the elements of a matrix are zero (i.e., $O_{ij} = 0$), it is termed a *null matrix*. Null matrices are usually denoted by $\mathbf{O}$ or $[O]$. An example of a 3×4 null matrix is given by

$$\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

MATRIX ALGEBRA

Equality

Matrices **A** and **B** are considered to be equal if they are of the same order and if their corresponding elements are identical (i.e., $A_{ij} = B_{ij}$). Consider, for example, matrices

$$\mathbf{A} = \begin{bmatrix} 6 & 2 \\ -7 & 8 \\ 3 & -9 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 6 & 2 \\ -7 & 8 \\ 3 & -9 \end{bmatrix}$$

Since both **A** and **B** are of order 3×2, and since each element of **A** is equal to the corresponding element of **B**, the matrices **A** and **B** are equal to each other; that is, $\mathbf{A} = \mathbf{B}$.

MATRIX ALGEBRA

Addition and Subtraction

Matrices can be added (or subtracted) only if they are of the same order. The addition (or subtraction) of two matrices $\mathbf{A}$ and $\mathbf{B}$ is carried out by adding (or subtracting) the corresponding elements of the two matrices. Thus, if $\mathbf{A}+\mathbf{B}=\mathbf{C}$, then $C_{ij}=A_{ij}+B_{ij}$; and if $\mathbf{A}-\mathbf{B}=\mathbf{D}$, then $D_{ij}=A_{ij}-B_{ij}$. The matrices $\mathbf{C}$ and $\mathbf{D}$ have the same order as matrices $\mathbf{A}$ and $\mathbf{B}$.

Calculate the matrices $\mathbf{C}=\mathbf{A}+\mathbf{B}$ and $\mathbf{D}=\mathbf{A}-\mathbf{B}$ if

$$\mathbf{A}=\begin{bmatrix} 6 & 0 \\ -2 & 9 \\ 5 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B}=\begin{bmatrix} 2 & 3 \\ 7 & 5 \\ -12 & -1 \end{bmatrix}$$

$$\mathbf{C}=\mathbf{A}+\mathbf{B}=\begin{bmatrix} (6+2) & (0+3) \\ (-2+7) & (9+5) \\ (5-12) & (1-1) \end{bmatrix}=\begin{bmatrix} 8 & 3 \\ 5 & 14 \\ -7 & 0 \end{bmatrix}$$

$$\mathbf{D}=\mathbf{A}-\mathbf{B}=\begin{bmatrix} (6-2) & (0-3) \\ (-2-7) & (9-5) \\ (5+12) & (1+1) \end{bmatrix}=\begin{bmatrix} 4 & -3 \\ -9 & 4 \\ 17 & 2 \end{bmatrix}$$

MATRIX ALGEBRA

Multiplication by a Scalar

The product of a scalar c and a matrix $\mathbf{A}$ is obtained by multiplying each element of the matrix $\mathbf{A}$ by the scalar c. Thus, if $c\mathbf{A} = \mathbf{B}$, then $B_{ij} = cA_{ij}$.

Calculate the matrix $\mathbf{B} = c\mathbf{A}$ if $c = -6$ and

$$\mathbf{A} = \begin{bmatrix} 3 & 7 & -2 \\ 0 & 8 & 1 \\ 12 & -4 & 10 \end{bmatrix}$$

$$\mathbf{B} = c\mathbf{A} = \begin{bmatrix} -6(3) & -6(7) & -6(-2) \\ -6(0) & -6(8) & -6(1) \\ -6(12) & -6(-4) & -6(10) \end{bmatrix} = \begin{bmatrix} -18 & -42 & 12 \\ 0 & -48 & -6 \\ -72 & 24 & -60 \end{bmatrix}$$

MATRIX ALGEBRA

Multiplication of Matrices

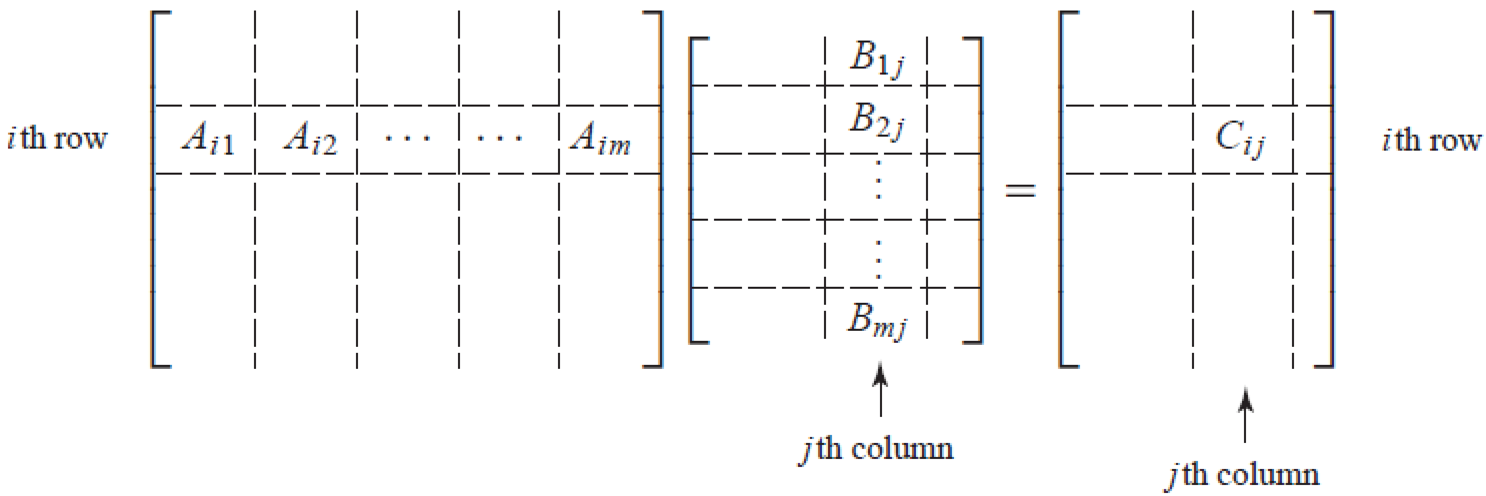
Two matrices can be multiplied only if the number of columns of the first matrix equals the number of rows of the second matrix. Such matrices are said to be *conformable* for multiplication. Consider, for example, the matrices

$$\mathbf{A} = \underset{3 \times 2}{\begin{bmatrix} 1 & 8 \\ 4 & -2 \\ -5 & 3 \end{bmatrix}} \quad \text{and} \quad \mathbf{B} = \underset{2 \times 2}{\begin{bmatrix} 6 & -7 \\ -1 & 2 \end{bmatrix}} \tag{2.3}$$

MATRIX ALGEBRA

$$\underset{(l \times m)}{\mathbf{A}} \quad \underset{(m \times n)}{\mathbf{B}} = \underset{(l \times n)}{\mathbf{C}} \tag{2.4}$$

equal



$$C_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j} + \cdots + A_{im}B_{mj}$$

MATRIX ALGEBRA

Calculate the product $\mathbf{C} = \mathbf{AB}$ of the matrices $\mathbf{A}$ and $\mathbf{B}$ given in Eq. (2.3).

$$\mathbf{C} = \mathbf{AB} = \underset{(3\times 2)}{\begin{bmatrix} 1 & 8 \\ 4 & -2 \\ -5 & 3 \end{bmatrix}} \underset{(2\times 2)}{\begin{bmatrix} 6 & -7 \\ -1 & 2 \end{bmatrix}} = \underset{(3\times 2)}{\begin{bmatrix} -2 & 9 \\ 26 & -32 \\ -33 & 41 \end{bmatrix}}$$

$C_{11} = 1(6) + 8(-1) = -2$

$C_{12} = 1(-7) + 8(2) = 9$

$C_{21} = 4(6) + (-2)(-1) = 26$

$C_{22} = 4(-7) -2(2) = -32$

$C_{31} = -5(6) + 3(-1) = -33$

$C_{32} = -5(-7) + 3(2) = 41$

MATRIX ALGEBRA

$$\begin{aligned}
A_{11}x_1 + A_{12}x_2 + A_{13}x_3 + A_{14}x_4 &= P_1 \\
A_{21}x_1 + A_{22}x_2 + A_{23}x_3 + A_{24}x_4 &= P_2 \\
A_{31}x_1 + A_{32}x_2 + A_{33}x_3 + A_{34}x_4 &= P_3 \\
A_{41}x_1 + A_{42}x_2 + A_{43}x_3 + A_{44}x_4 &= P_4
\end{aligned}$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$\mathbf{Ax} = \mathbf{P}$$

MATRIX ALGEBRA

Matrix multiplication is generally not commutative; that is,

$$\boxed{\mathbf{AB} \neq \mathbf{BA}}$$

Calculate the products **AB** and **BA** if

$$\mathbf{A} = \begin{bmatrix} 1 & -8 \\ -7 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 6 & -3 \\ 4 & -5 \end{bmatrix}$$

Are the products **AB** and **BA** equal?

$$\mathbf{AB} = \begin{bmatrix} 1 & -8 \\ -7 & 2 \end{bmatrix}\begin{bmatrix} 6 & -3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} -26 & 37 \\ -34 & 11 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 6 & -3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 1 & -8 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 27 & -54 \\ 39 & -42 \end{bmatrix}$$

Comparing products **AB** and **BA**, we can see that $\mathbf{AB} \neq \mathbf{BA}$.

MATRIX ALGEBRA

Matrix multiplication is associative and distributive, provided that the sequential order in which the matrices are to be multiplied is maintained. Thus,

$$\mathbf{ABC} = (\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC}) \tag{2.11}$$

and

$$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC} \tag{2.12}$$

MATRIX ALGEBRA

The product of any matrix $\mathbf{A}$ and a conformable null matrix $\mathbf{O}$ equals a null matrix; that is,

$$\mathbf{AO} = \mathbf{O} \qquad \text{and} \qquad \mathbf{OA} = \mathbf{O} \tag{2.13}$$

For example,

$$\begin{bmatrix} 2 & -4 \\ -6 & 8 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The product of any matrix $\mathbf{A}$ and a conformable unit matrix $\mathbf{I}$ equals the original matrix $\mathbf{A}$; thus,

$$\mathbf{AI} = \mathbf{A} \quad \text{and} \quad \mathbf{IA} = \mathbf{A} \tag{2.14}$$

For example,

$$\begin{bmatrix} 2 & -4 \\ -6 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -6 & 8 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ -6 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -6 & 8 \end{bmatrix}$$

MATRIX ALGEBRA

Transpose of a Matrix

The *transpose* of a matrix is obtained by interchanging its corresponding rows and columns. The transposed matrix is commonly identified by placing a superscript T on the symbol of the original matrix. Consider, for example, a 3×2 matrix

$$\mathbf{B} = \underset{3 \times 2}{\begin{bmatrix} 2 & -4 \\ -5 & 8 \\ 1 & 3 \end{bmatrix}}$$

The transpose of $\mathbf{B}$ is given by

$$\mathbf{B}^T = \underset{2 \times 3}{\begin{bmatrix} 2 & -5 & 1 \\ -4 & 8 & 3 \end{bmatrix}}$$

$$\boxed{(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T}$$

$$(\mathbf{ABC})^T = \mathbf{C}^T\mathbf{B}^T\mathbf{A}^T$$

MATRIX ALGEBRA

Inverse of a Square Matrix

The inverse of a square matrix $\mathbf{A}$ is defined as a matrix $\mathbf{A}^{-1}$ with elements of such magnitudes that the product of the original matrix $\mathbf{A}$ and its inverse $\mathbf{A}^{-1}$ equals a unit matrix $\mathbf{I}$; that is,

$$\boxed{\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}} \qquad \textbf{(2.17)}$$

The operation of inversion is defined only for square matrices, with the inverse of such a matrix also being a square matrix of the same order as the original matrix. A procedure for determining inverses of matrices will be presented in the next section.

$$\mathbf{A}\mathbf{x} = \mathbf{P}$$

$$\mathbf{A}^{-1}\mathbf{A}\mathbf{x} = \mathbf{A}^{-1}\mathbf{P}$$

As $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$ and $\mathbf{I}\mathbf{x} = \mathbf{x}$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{P}$$

MATRIX ALGEBRA

Partitioning of Matrices

In many applications, it becomes necessary to subdivide a matrix into a number of smaller matrices called *submatrices*. The process of subdividing a matrix into submatrices is referred to as *partitioning*. For example, a 4×3 matrix $\mathbf{B}$ is partitioned into four submatrices by drawing horizontal and vertical dashed partition lines:

$$\mathbf{B} = \left[\begin{array}{rr|r} 2 & -4 & -1 \\ -5 & 7 & 3 \\ 8 & -9 & 6 \\ \hline 1 & 3 & 8 \end{array}\right] = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \tag{2.18}$$

in which the submatrices are

$$\mathbf{B}_{11} = \begin{bmatrix} 2 & -4 \\ -5 & 7 \\ 8 & -9 \end{bmatrix} \qquad \mathbf{B}_{12} = \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix}$$

$$\mathbf{B}_{21} = [1 \quad 3] \qquad \mathbf{B}_{22} = [8]$$

MATRIX ALGEBRA

Matrix operations (such as addition, subtraction, and multiplication) can be performed on partitioned matrices in the same manner as discussed previously by treating the submatrices as elements—provided that the matrices are partitioned in such a way that their submatrices are conformable for the particular operation. For example, suppose that the 4×3 matrix $\mathbf{B}$ of Eq. (2.18) is to be postmultiplied by a 3×2 matrix $\mathbf{C}$, which is partitioned into two submatrices:

$$\mathbf{C} = \left[\begin{array}{rr} 9 & -6 \\ 4 & 2 \\ \hdashline -3 & 1 \end{array}\right] = \begin{bmatrix} \mathbf{C}_{11} \\ \mathbf{C}_{21} \end{bmatrix} \tag{2.19}$$

The product $\mathbf{BC}$ is expressed in terms of submatrices as

$$\mathbf{BC} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{C}_{11} \\ \mathbf{C}_{21} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11}\mathbf{C}_{11} + \mathbf{B}_{12}\mathbf{C}_{21} \\ \mathbf{B}_{21}\mathbf{C}_{11} + \mathbf{B}_{22}\mathbf{C}_{21} \end{bmatrix} \tag{2.20}$$

It is important to realize that matrices $\mathbf{B}$ and $\mathbf{C}$ have been partitioned in such a way that their corresponding submatrices are conformable for multiplication; that is, the orders of the submatrices are such that the products $\mathbf{B}_{11}\mathbf{C}_{11}$, $\mathbf{B}_{12}\mathbf{C}_{21}$, $\mathbf{B}_{21}\mathbf{C}_{11}$, and $\mathbf{B}_{22}\mathbf{C}_{21}$ are defined. It can be seen from Eqs. (2.18) and (2.19) that this is achieved by partitioning the rows of the second matrix $\mathbf{C}$ of the product $\mathbf{BC}$ in the same way that the columns of the first matrix $\mathbf{B}$ are partitioned. The products of the submatrices are:

MATRIX ALGEBRA

$$\mathbf{B}_{11}\mathbf{C}_{11} = \begin{bmatrix} 2 & -4 \\ -5 & 7 \\ 8 & -9 \end{bmatrix} \begin{bmatrix} 9 & -6 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -20 \\ -17 & 44 \\ 36 & -66 \end{bmatrix}$$

$$\mathbf{B}_{12}\mathbf{C}_{21} = \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix} [-3 \quad 1] = \begin{bmatrix} 3 & -1 \\ -9 & 3 \\ -18 & 6 \end{bmatrix}$$

$$\mathbf{B}_{21}\mathbf{C}_{11} = [1 \quad 3] \begin{bmatrix} 9 & -6 \\ 4 & 2 \end{bmatrix} = [21 \quad 0]$$

$$\mathbf{B}_{22}\mathbf{C}_{21} = [8][-3 \quad 1] = [-24 \quad 8]$$

By substituting the numerical values of the products of submatrices into Eq. (2.20), we obtain

$$\mathbf{BC} = \begin{bmatrix} \begin{bmatrix} 2 & -20 \\ -17 & 44 \\ 36 & -66 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -9 & 3 \\ -18 & 6 \end{bmatrix} \\ [21 \quad 0] + [-24 \quad 8] \end{bmatrix} = \begin{bmatrix} 5 & -21 \\ -26 & 47 \\ 18 & -60 \\ -3 & 8 \end{bmatrix}$$

MATRIX ALGEBRA

1. A matrix is defined as a rectangular array of quantities (elements) arranged in rows and columns. The size of a matrix is measured by its number of rows and columns, and is referred to as its order.
2. Two matrices are considered to be equal if they are of the same order, and if their corresponding elements are identical.
3. Two matrices of the same order can be added (or subtracted) by adding (or subtracting) their corresponding elements.
4. The matrix multiplication $\mathbf{AB} = \mathbf{C}$ is defined only if the number of columns of the first matrix $\mathbf{A}$ equals the number of rows of the second matrix $\mathbf{B}$.

MATRIX ALGEBRA

5. The transpose of a matrix is obtained by interchanging its corresponding rows and columns. If $\mathbf{C}$ is a symmetric matrix, then $\mathbf{C}^T = \mathbf{C}$. Another useful property of matrix transposition is that

$$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T \qquad \textbf{(2.15)}$$

6. A matrix can be differentiated (or integrated) by differentiating (or integrating) each of its elements.

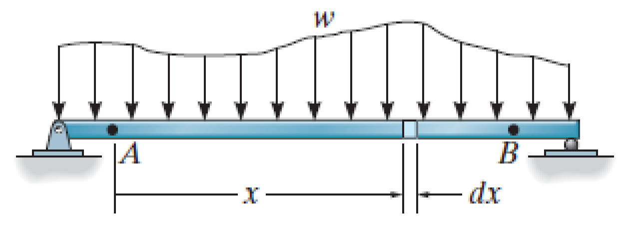
7. The inverse of a square matrix $\mathbf{A}$ is defined as a matrix $\mathbf{A}^{-1}$ which satisfies the relationship:

$$\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \qquad \textbf{(2.17)}$$

8. If the inverse of a matrix equals its transpose, the matrix is called an orthogonal matrix.

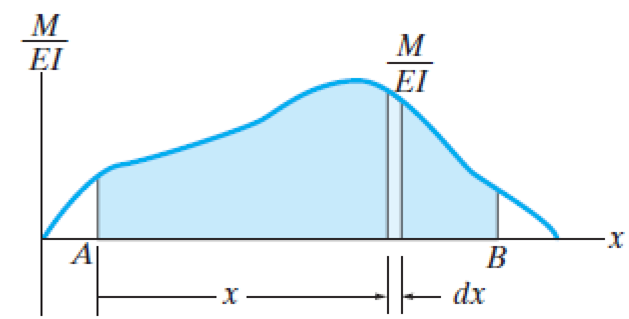
Moment-Area Theorem

Theorem : The change in slope between any two points on the elastic curve equals the area of the *M/EI* diagram between these two points.



(a)

$$\theta_{B/A} = \int_A^B \frac{M}{EI}\,dx$$



(b)

This equation forms the basis for the first moment-area theorem.

elastic curve
(c)

Study of Indeterminacies

Statically indeterminacy:

When all the forces in a structure can be determined strictly from equilibrium equations, the structure is referred to as ***statically determinate***. Structures having more unknown forces than available equilibrium equations are called ***statically indeterminate***

The degree of static indeterminacy is always defined as the difference between the number of unknown forces and the number of equilibrium equations available to solve for the unknowns. These extra forces are called redundant

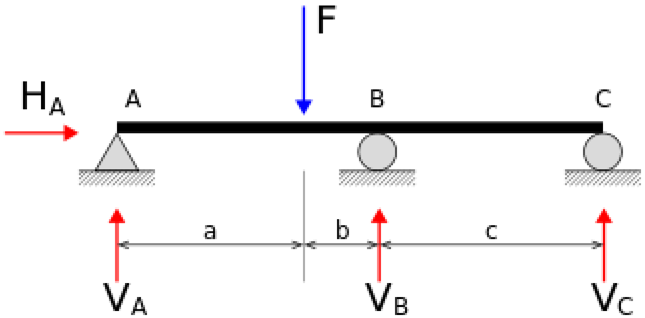
Degree of static indeterminacy (DOI) = Total number of unknown forces - Number of independent equations of equilibrium

Based on Newton's laws of motion, the equilibrium equations available for a two-dimensional body are

- : $\sum \vec{F} = 0$ the sum of the forces acting on the body equals zero. This translates to
  Σ F_x = 0: the sum of the horizontal components of the forces equals zero;
  Σ F_y = 0: the sum of the vertical components of forces equals zero;

- : $\sum \vec{M} = 0$ the sum of the moments (about an arbitrary point) of all forces equals zero.

Since there are four unknown forces (or *variables*) (V_A, V_B, V_C and H_A) but only three equilibrium equations,



The structure is therefore classified as *statically indeterminate to one degree*.

Study of Indeterminacies

Kinematic Indeterminacy:

Another form of the indeterminacy of a structure is expressed in terms of its ***degrees of freedom***; this is known as the ***kinematic indeterminacy*** of a structure and is of particular relevance in the stiffness method of analysis where the unknowns are the displacements.

A simple approach to calculating the kinematic indeterminacy of a structure is to sum the degrees of freedom of the nodes and then subtract those degrees of freedom that are prevented by constraints such as support points.

It is therefore important to remember that in plane structures each node possess three degrees of freedom.

Study of Indeterminacies

Kinematic Indeterminacy:

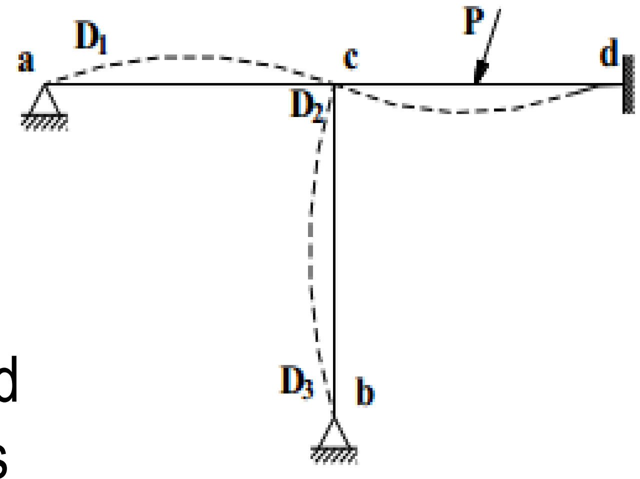
Consider a propped cantilever beam shown in Fig. Usually, the axial rigidity of the beam is so high that the change in its length along axial direction may be neglected. The displacements at a fixed support are zero. Hence, for a propped cantilever beam we have to evaluate only rotation at B and this is known as the kinematic indeterminacy of the structure.

(a) Propped Cantilever Beam

Study of Indeterminacies

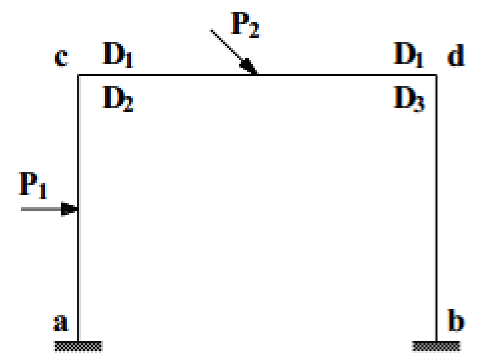
Kinematic Indeterminacy:

The frame given in the figure (a) has three degrees of kinematic indeterminacy. The rotations D1, D2 and D3 at joints a, b and c.



(a)

The frame given in the figure (b), joints c and d have a horizontal displacement D1,point c has horizontal displacement D1 and rotation D2. While point d has horizontal displacement D1 and rotation D3. Notice that the horizontal displacement at point c and d are equal. The fixation at supports a and b prevent any translation or rotation.



(b)